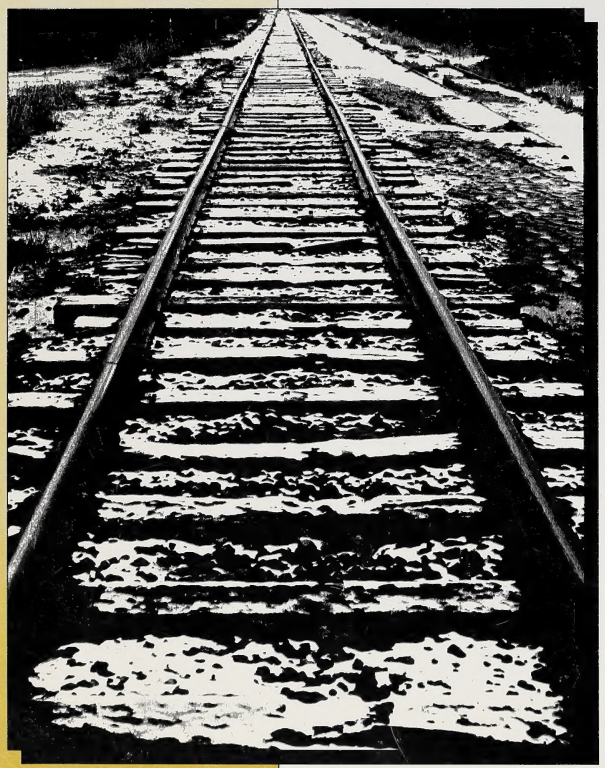
MATHEMATICS



8

MODULE 6
MEASUREMENT AND GEOMETRY

Distance Learning

Alberta EDUCATION

Mathematics 8

Module 6: Measurement and Geometry

MODULE BOOKLET

Mathematics 8
Student Module
Module 6
Measurement and Geometry

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Welcome to Mathematics 8!

We hope you'll enjoy your study of ***Measurement and Geometry****.*

To make your learning a bit easier, a teacher will help guide you through the materials.

So whenever you see this icon,

turn on your audiocassette and listen.

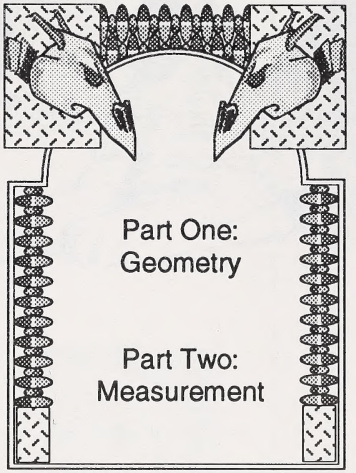
CONTENTS AT A GLANCE

What Lies Ahead

The Module Introduction will give you an overview of Module 6.

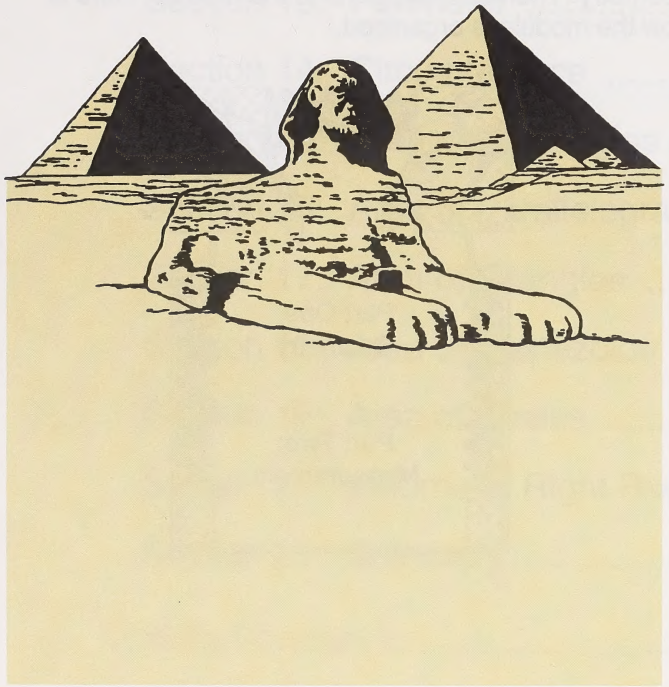
Working Together

In Module 6 you will be working with measurement and geometry. There are two parts to the module. Here is how the module is organized.

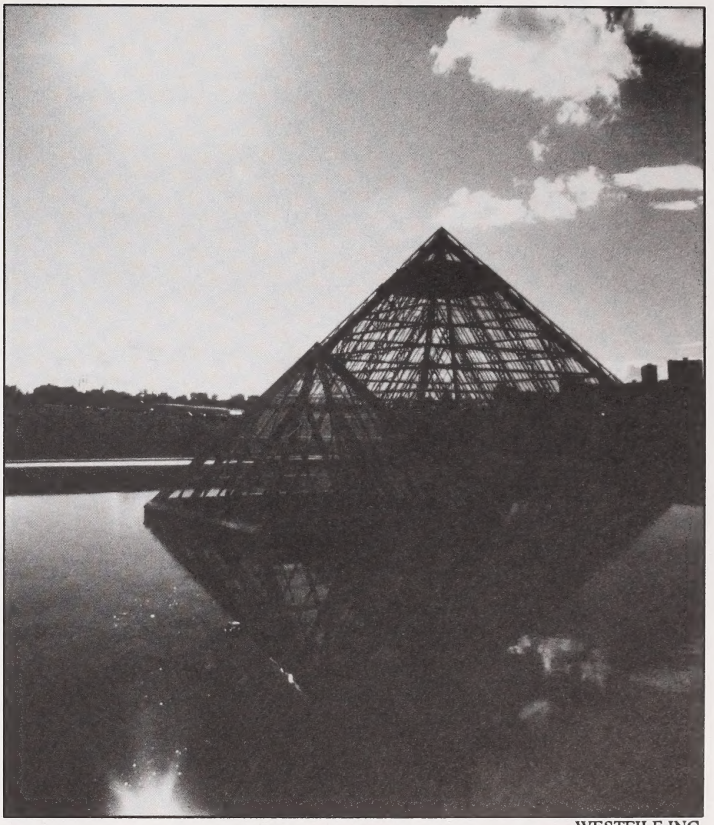


Geometry is one of the oldest branches of mathematics. The word geometry comes from the Greek *geo* meaning *earth* and *metros* meaning *to measure*.

Early Egyptian architects used measurement and geometry to design the pyramids.



Modern architects use measurement and geometry to design modern buildings.



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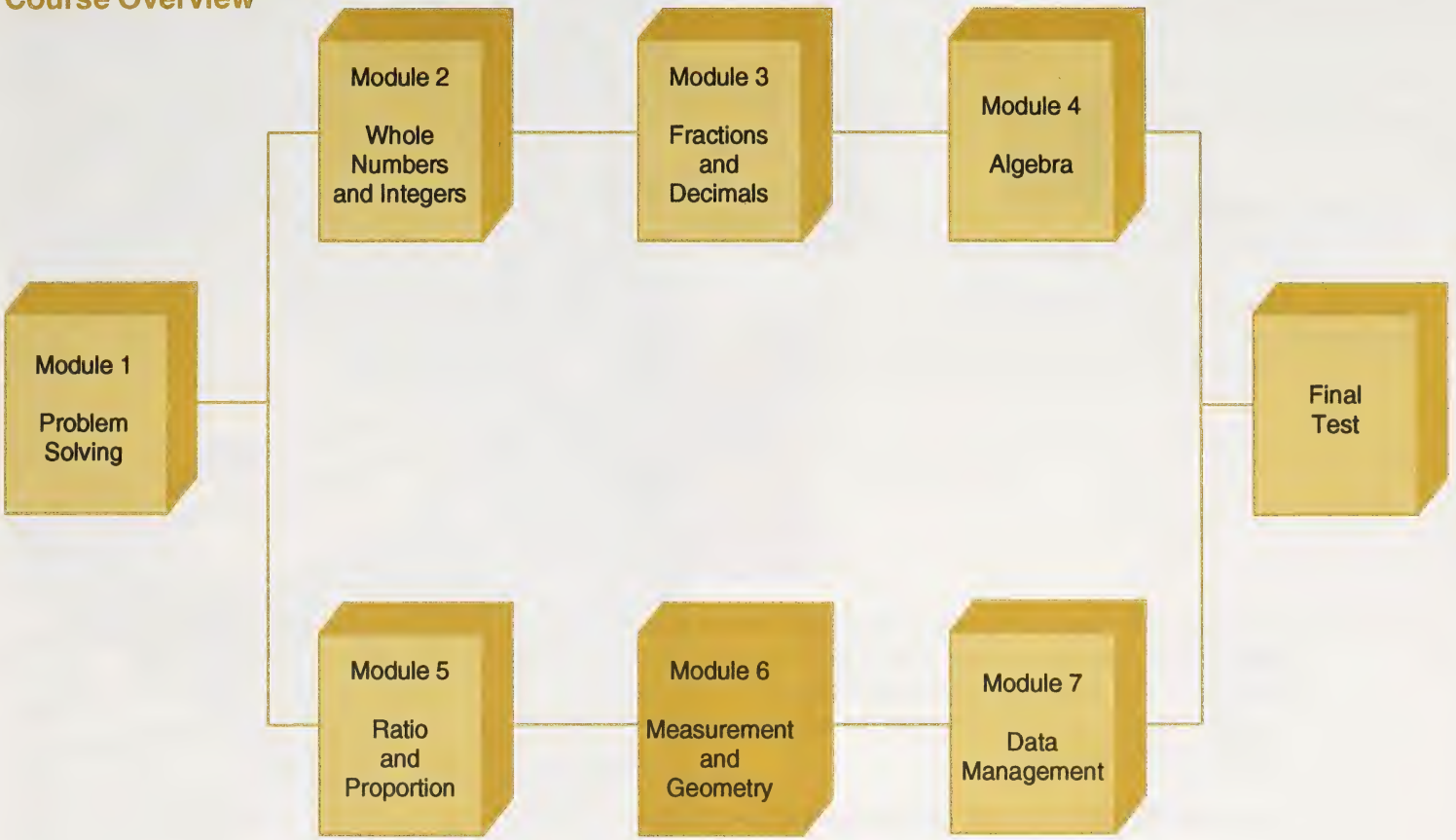
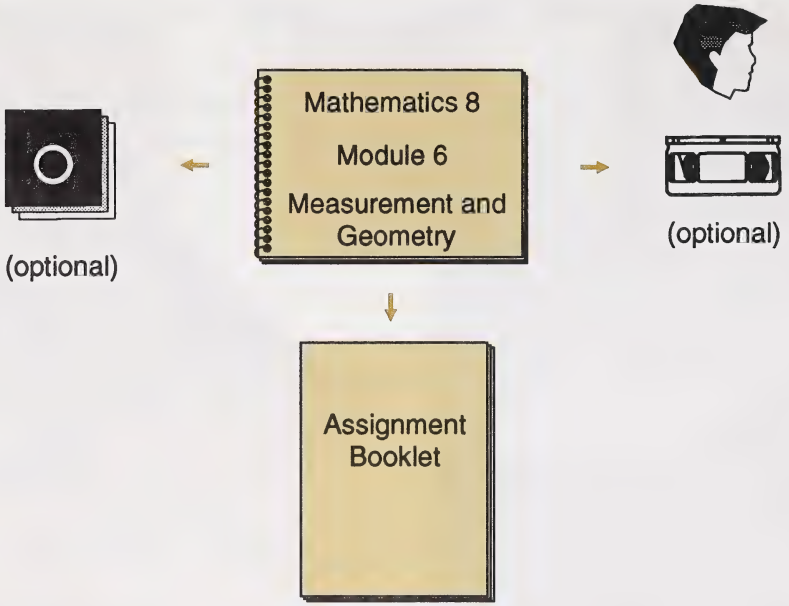
Course Overview



Mathematics 8 has seven modules and a final supervised test. This module booklet is part of Module 6.

Module 6 Components

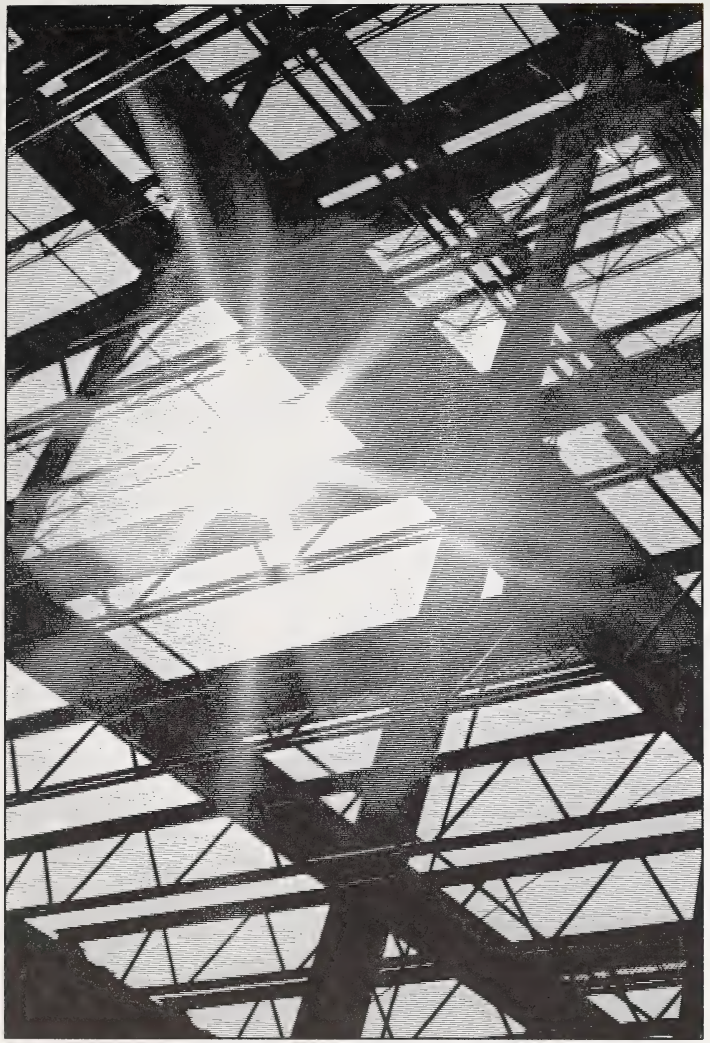


This module booklet will give you instruction and practice in learning mathematical skills and words. It will also direct you to the other components of the module. The computer and video activities in this booklet are optional; there are print alternatives. You should see your learning facilitator to check your answers to the activities in this booklet. This booklet is not to be submitted for a grade.

Your mark on this module will be determined by your work in the assignment booklet.

Take time to preview this module booklet before beginning Section 1.

PART ONE



Sections 1 to 10 deal with geometry.

In Grade 7 you learned about motion geometry and investigated slides, flips, turns, congruence, similarity, and symmetry. You also made geometrical designs using tiling, a compass, and a computer language called LOGO.

In Grade 8 you will investigate the properties of angles, lines, polygons, and right rectangular prisms.

Much of the geometry that you will learn in Grade 8 was studied by Euclid, a famous Greek mathematician who lived from approximately 330 BC to 270 BC.

What Lies Ahead

In this section you will review these concepts.

- slides, flips, and turns
- congruent figures
- similar figures
- flip and turn symmetry
- tiling
- tesselations
- geometric designs

Working Together

To begin this part of Module 6, you will review your knowledge of concepts taught in Grade 7.

The following review will help you and your learning facilitator discover your strengths and weaknesses.

Review

Space for Your Work

1. What transformations (slides, turns, or flips) are suggested by the following activities?

   a. moving furniture into a new house

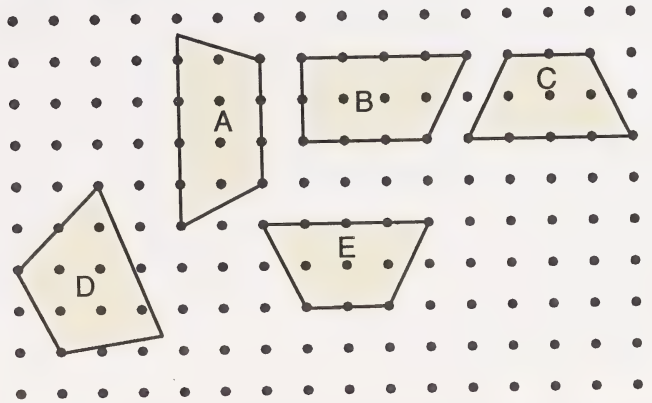
   b. playing both sides of a record

   c. resetting your watch

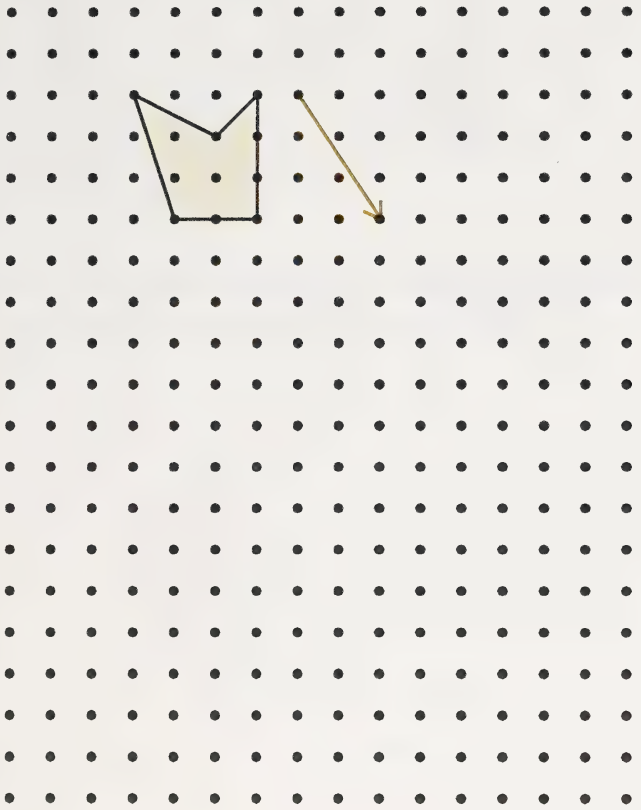
   d. raising a flag up a flag pole

   e. playing chess or checkers

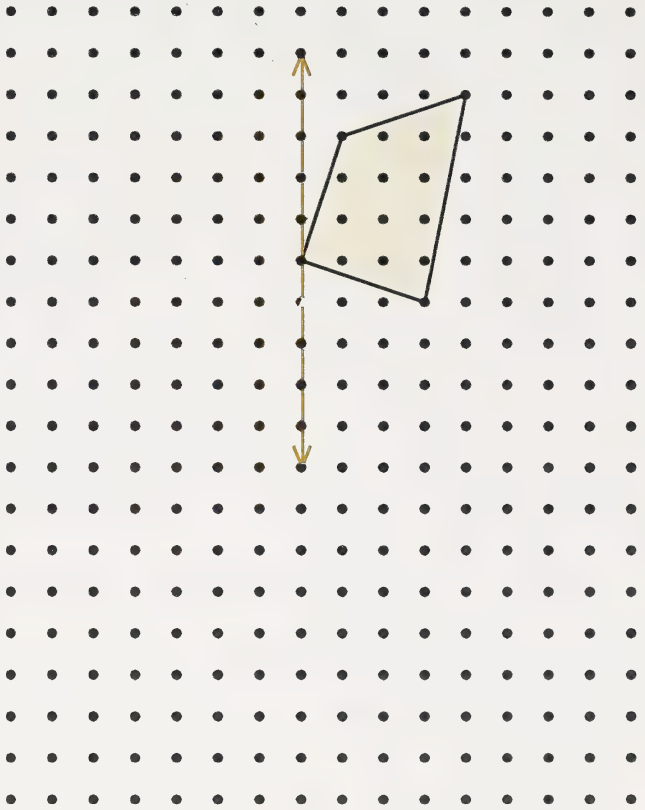
2. Which of the following figures are congruent?



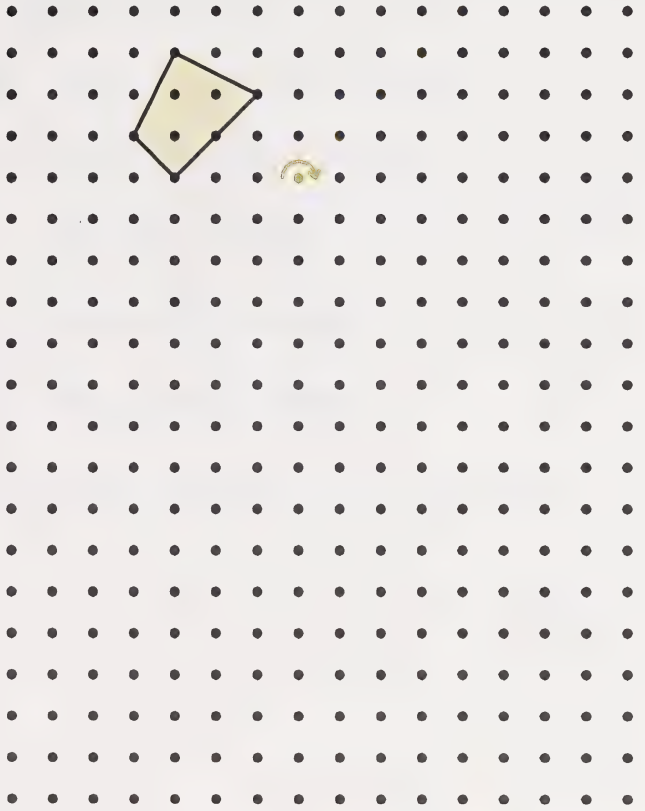
3. Draw the slide image for the given slide arrow. You may use the tracing paper provided in the Appendix.



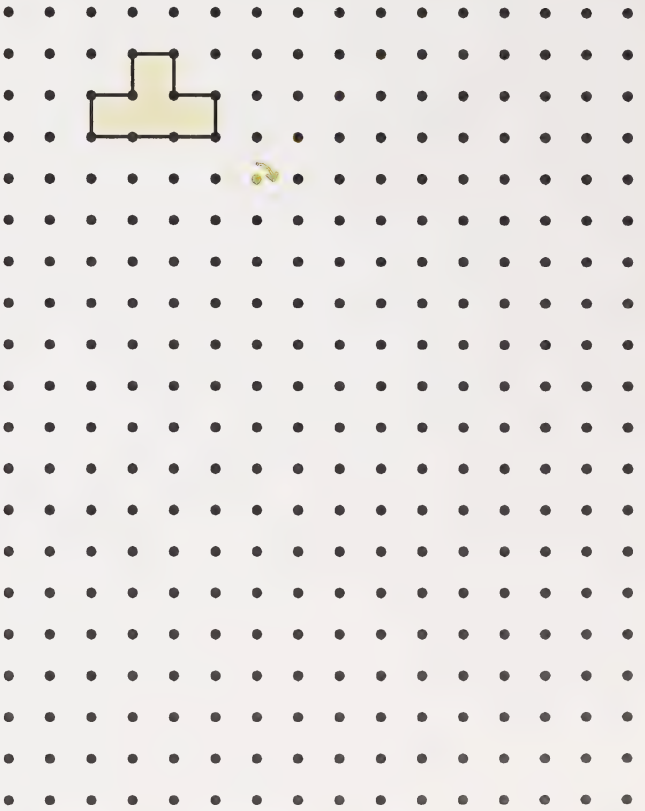
4. Draw the flip image for the given flip line. You may use the tracing paper provided in the Appendix.



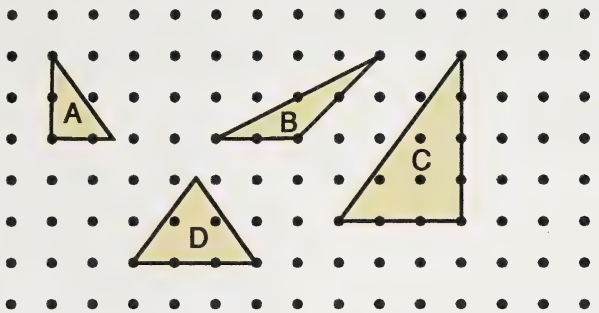
5. Draw the half-turn image for the given turn centre. You may use the tracing paper provided in the Appendix.



6. Draw the quarter-turn image for the given turn centre. You may use the tracing paper provided in the Appendix.

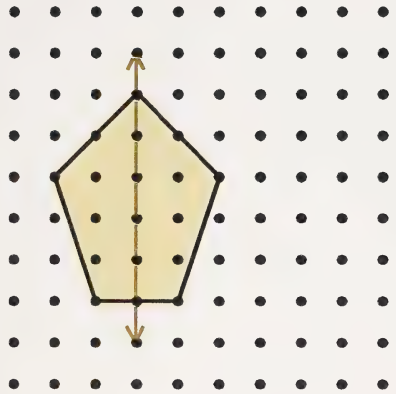


7. Which of the following figures are similar?



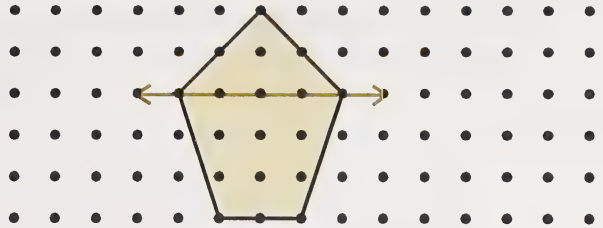
8. For each of the following figures, indicate whether or not the lines shown are lines of symmetry.

a.



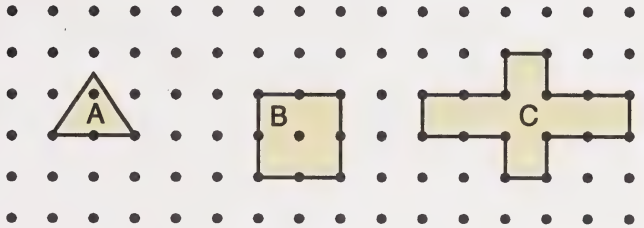
Space for Your Work

b.

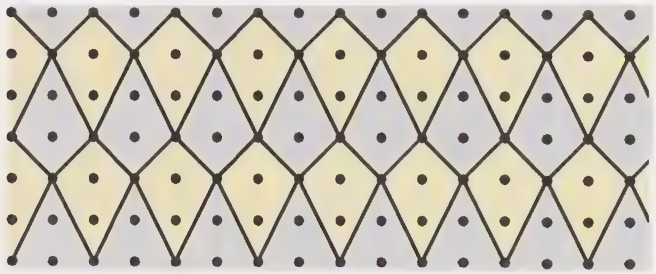


Space for Your Work

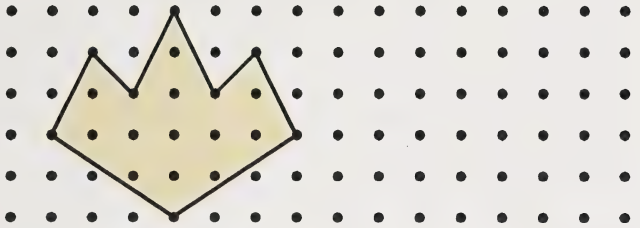
9. Which figure has a turn order of 2?



10. How many different shapes are used to make this tiling pattern?



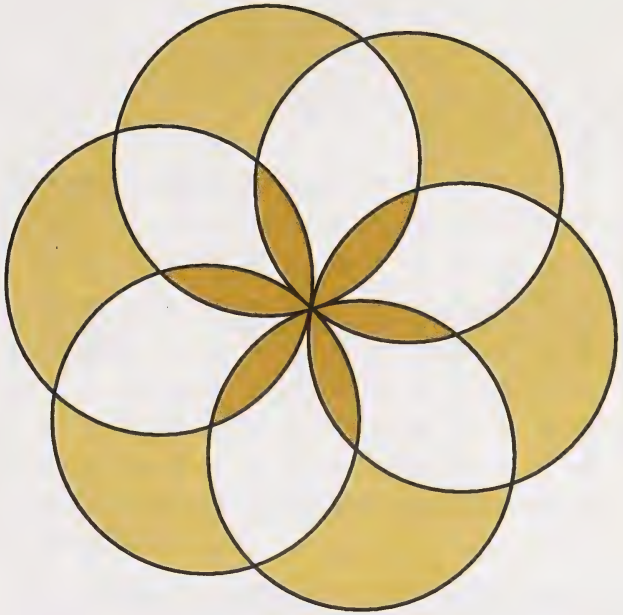
11. Create a tesselation with this shape. Use the dot paper provided at the right.



Space for Your Work

11.

12. Construct this design with a compass. You can make the design larger or smaller if you wish.



Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

What Lies Ahead

In this section you will pretest these skills.

- interpreting lines, rays, and line segments
- interpreting polygons
- classifying angles
- classifying lines
- classifying polygons according to number of sides and angles
- classifying triangles
- classifying quadrilaterals
- interpreting right rectangular prisms and cubes

Working Together

The following pretest will help you and your learning facilitator determine your strengths and weaknesses in geometry.

Pretest

Space for Your Work

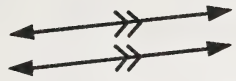
1. Choose the best word to describe each of the following (e.g., ray, parallel, perpendicular, line segment, etc.).

a.

b.

c.

d.

e. 

f. 

2. Choose the best word for each of the following angles (e.g., acute, obtuse, etc.).

a. 

b. 

Space for Your Work

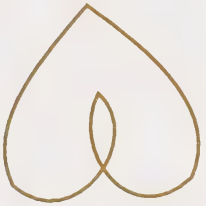
c.

Space for Your Work

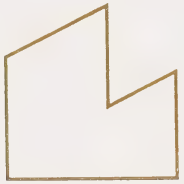
d.

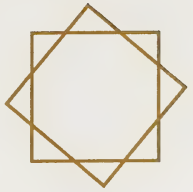
3. Which of the following figures are polygons? Explain why.

a.



b.



c. 

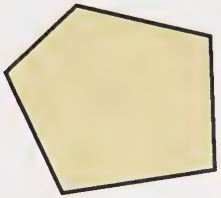
d. 

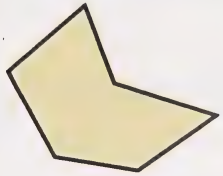
e. 

Space for Your Work

4. Choose the name that shows the number of sides and angles for the following figures.

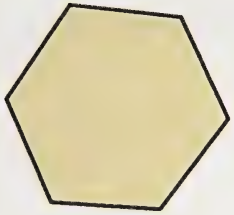
Space for Your Work

a. 

b. 

c. 

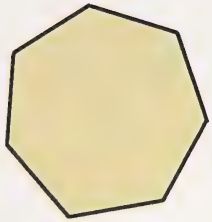
d.



Space for Your Work

e.

f.

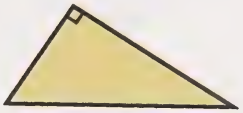


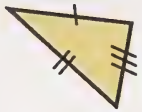
5. Choose the best word to describe each of the following triangles.

Space for Your Work

a. 

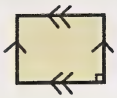
b. 

c. 

d. 

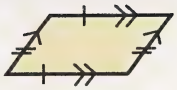
6. Choose the best name for each of the following polygons.

Space for Your Work

a. 

b. 

c. 

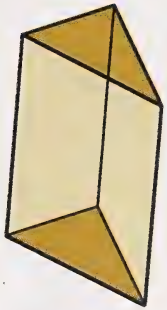
d. 

e. 

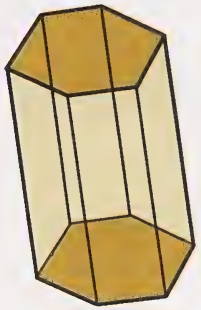
7. Which of the following figures are right rectangular prisms?

Space for Your Work

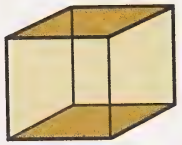
a.



b.



c.



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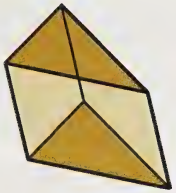

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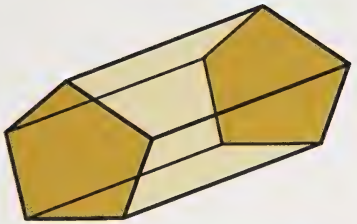

f.

Space for Your Work

8. Which of the right rectangular prisms in Question 7 is a cube?

Space for Your Work

9. a. How many faces does a right rectangular prism have?

   b. How many faces edges does a right rectangular prism have?

   c. How many vertices does a right rectangular prism have?

See your learning facilitator to check your answers and to receive further instructions.

What Lies Ahead

In this section you will learn these skills.

- naming points, line segments, and curves
- indicating congruent segments

In this section you will learn these terms.

- point
- endpoint
- midpoint
- line segment
- curve
- closed curve
- non-simple closed curve
- simple closed curve
- polygon
- convex polygon
- concave polygon
- vertex
- congruent sides
- adjacent sides
- non-adjacent sides

Working Together

Mathematics is like many other fields — it becomes easier if a common language is known.

Carpenters talk about hammers, planes, saws, drills, and screwdrivers.

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Mathematicians talk about points, curves, and line segments.

Point

A point is a single location in space. It has no length, width, or depth.

A point is represented by a dot and labelled with a letter.

Example

• P

You say *point P.*

You write P.

The positions of different points are represented by dots labelled with different letters.

Example

• P • Q

• R

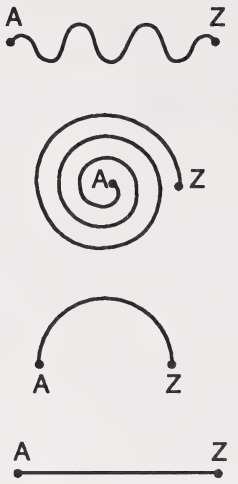
You say *point P*, *point Q*, and *point R.*

You write P, Q, R.

Curves

A curve is a connected set of points. Here are some examples of curves.

Examples



Line Segment

A line segment is a set of connected points that lie in a straight path.

A line segment is represented by joining two points labelled with letters.

Example

You say *line segment AZ* or *line segment ZA.*

You may also say *segment AZ* or *segment ZA.*

You write $\overline{AZ}$ or $\overline{ZA}$.

Endpoint

Each point at the end of a segment is called the endpoint.

Example

The endpoints of $\overline{AZ}$ are A and Z.

Midpoint

A point that divides a segment into two equal parts is called the midpoint.

Example

M is the midpoint of $\overline{AZ}$.

The congruent parts of the segment may be indicated with slash marks.

Introductory Activities

Space for Your Work

1. Draw any design without lifting your pencil from the paper. You will have drawn a curve.

2. Which of the following curves can you trace without lifting your pencil from the paper? You may retrace or cross over a part you have already drawn.

   a.

   b.

   c.

   d.

   e.

3. Which of the curves in Question 2 did not have endpoints?

4. Do either part a. or part b. of this question.

   a. Use a geoboard and elastics. Make as many different shapes as you can by joining five points with segments.

   b. Use the dot paper from the Appendix. Make as many different shapes as you can by joining five points with segments.

5. How are the shapes that you made in Question 4 different?

Space for Your Work

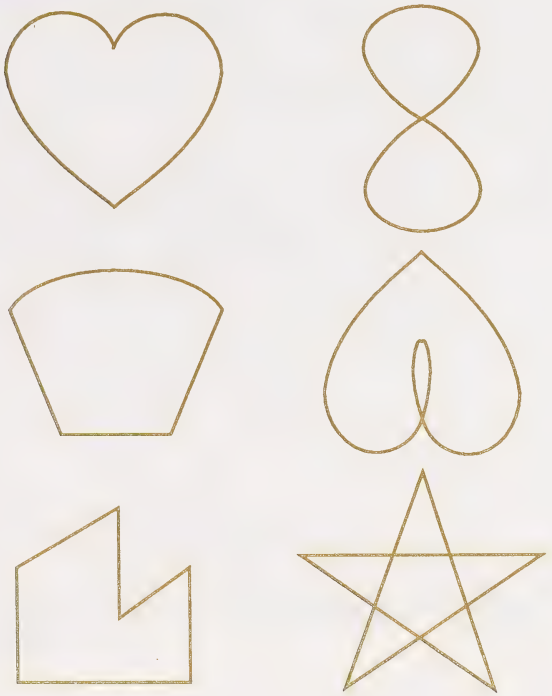
See your learning facilitator to check your answers and to receive further instructions.

Working Together

Closed Curves

A **closed curve** is a collection of points without any endpoints.

Examples



Closed curves with crossovers are called **non-simple closed curves**.

Examples

Closed curves without any crossovers are called **simple closed curves**.

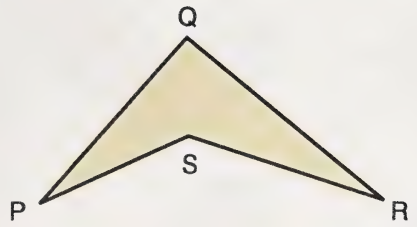
Examples

Simple closed curves have an inside and an outside.

Polygons

A **polygon** is a simple closed curve made up of line segments.

Example



The segments which make up the polygon are called the **sides** of the polygon.

In the example, $\overline{PQ}$, $\overline{QR}$, $\overline{SR}$, and $\overline{PS}$ are the sides of the polygon.

The sides of a polygon are referred to as **adjacent** and **non-adjacent**. Adjacent sides are next to each other. Non-adjacent sides are not next to each other.

In the example, $\overline{PQ}$ and $\overline{QR}$ are adjacent sides. $\overline{PQ}$ and $\overline{SR}$ are non-adjacent sides.

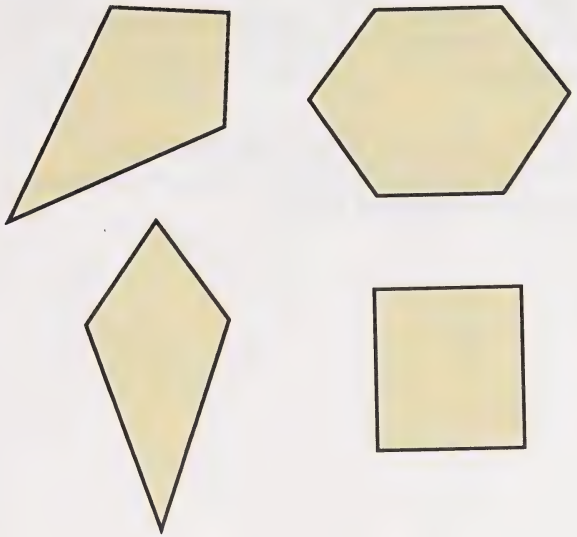
The point at which two adjacent sides meet is called a **vertex** of the polygon. The plural of vertex is **vertices**. In the example, P, Q, R, and S are vertices.

You use the vertices to name the polygon. You can say *polygon PQRS*, *polygon QRSP*, *polygon RSPQ*, or *polygon SPQR*. Notice that the points are named in a clockwise order.

You can also say *polygon PSRQ*, *polygon SRQP*, *polygon RQPS*, or *polygon QPSR*. Here the points are named in a counterclockwise order.

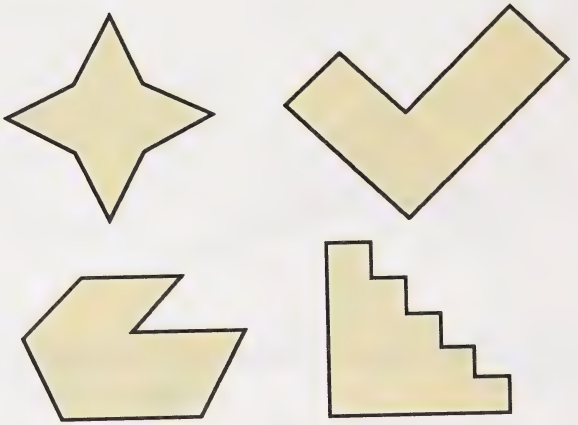
If all the sides of a polygon turn outward, the polygon is called a **convex polygon**.

Examples



If some of the sides turn inward, the polygon is called a **concave polygon**.

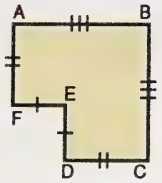
Examples



Congruent Sides

Congruent sides in a figure are indicated with slash marks. Congruent sides are the same length.

Example



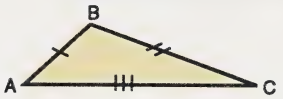
$\overline{FE} \cong \overline{ED}$

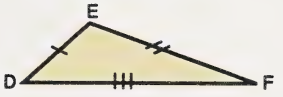
$\overline{AF} \cong \overline{DC}$

$\overline{AB} \cong \overline{BC}$

≅ means *is congruent to.*

Congruent sides in two figures are also indicated with slash marks.





$\overline{AB} \cong \overline{DE}$

$\overline{AC} \cong \overline{DF}$

$\overline{CB} \cong \overline{FE}$

Practice Activities

Space for Your Work

1. Is each of the following curves a polygon? Why or why not?

a. 

b. 

c. 

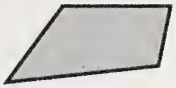
d. 

e. 

f. 

g. 

h. 

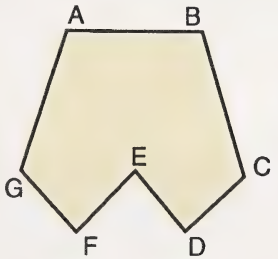
i. 

j. 

Space for Your Work

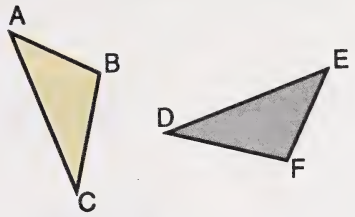
2. Which of the polygons in Question 1 are convex polygons?

3. Name the sides adjacent to $\overline{AB}$ in this polygon.

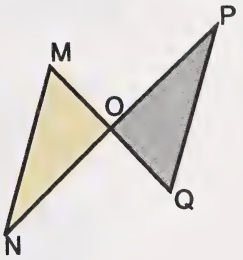


4. Complete the blanks at the right. Use slash marks to indicate congruent sides on the figures.

a.



b.



Space for Your Work

a. $\overline{AB} \cong$ _____

$\overline{AC} \cong$ _____

$\overline{BC} \cong$ _____

b. $\overline{MO} \cong$ _____

$\overline{MN} \cong$ _____

$\overline{NO} \cong$ _____

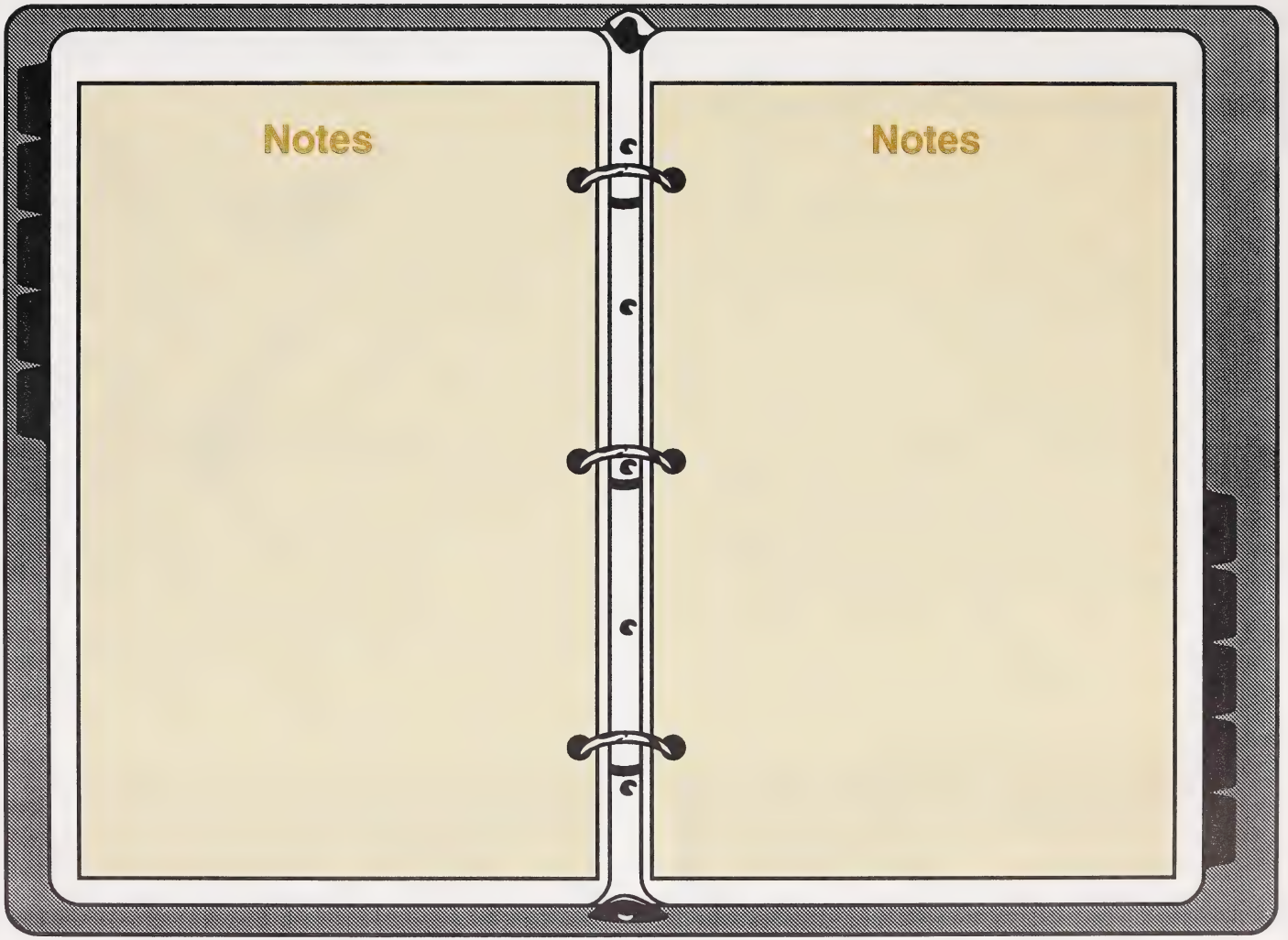
See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

Space for Your Work

Connect all the points to the right with only four line segments. Do not lift your pencil and do not retrace any lines.

See your learning facilitator to check your answers and to receive further instructions.


Notes
Notes

What Lies Ahead

In this section you will learn these skills.

- naming angles
- measuring angles
- classifying angles
- indicating congruent angles

In this section you will also learn these terms.

- rays
- angles
- degrees
- turns
- acute angle
- right angle
- straight angle
- obtuse angle
- reflex angle
- congruent angle

Working Together

In the previous section you learned about points, lines, and line segments. In this section you will learn about rays and angles.

Rays

A ray is a collection of points in a straight path. A ray continues in one direction forever.

Example

The arrowhead indicates that the ray continues in this direction forever.

You would say *ray PR.*

You would write $\overrightarrow{PR}$.

Note

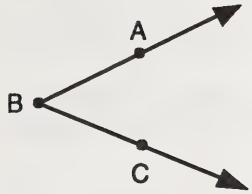
When naming a ray, the endpoint is always given first.

A lighthouse beam suggests a ray.

Angles

An angle is formed by two rays.

Example



This angle is formed by $\overrightarrow{BA}$ and $\overrightarrow{BC}$.

The common endpoint is called the **vertex**.

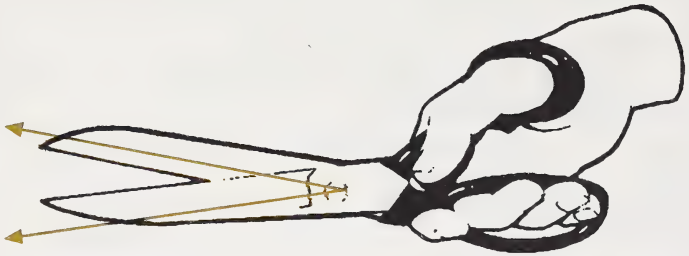
You would say *angle ABC* or *angle CBA.*

You would write $\angle ABC$ or $\angle CBA$.

Note

When naming an angle, the vertex is always named in the middle.

The blades of a jackknife or a pair of scissors may suggest an angle.

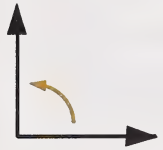


Turns

Angles can be measured in turns.

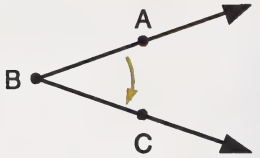
Examples

- This angle is a quarter turn.



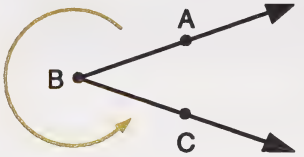
- This angle is a half turn.

Every angle has two different turns associated with it — a smaller turn and a larger turn.



Here $\overrightarrow{BA}$ is the initial ray, and $\overrightarrow{BC}$ is the final ray.

This angle is less than a quarter turn.



Here $\overrightarrow{BA}$ is the initial ray and $\overrightarrow{BC}$ is the final ray.

This angle is almost a full turn.

Note

The smaller turn is intended if the turn is not indicated.

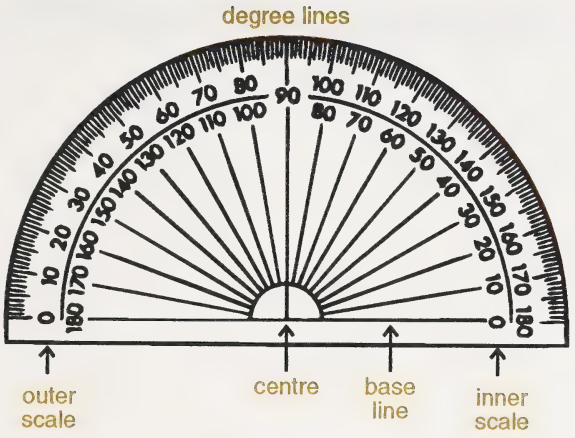
Degrees

Angles can be measured in degrees. The symbol ° is used for degrees. There are 360° in a full turn, 180° in a half turn, and 90° in a quarter turn.

Tools for Measuring Angles

A protractor is used to measure angles in degrees. It is marked off into 180 degree lines.

The **centre** of the protractor is the starting point for all degree lines. The 0° line and the 180° line is called the **base line**. The centre and the base line are used to position the protractor properly when measuring angles.



The protractor has an **outer scale** and an **inner scale**. You should be careful to read the appropriate scale when measuring angles.

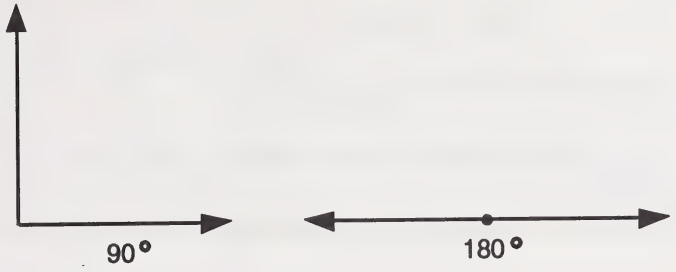
Video Activity

Watch *SOLVE IT: Measuring Angles.*

If you cannot view the video, read the notes in this section.

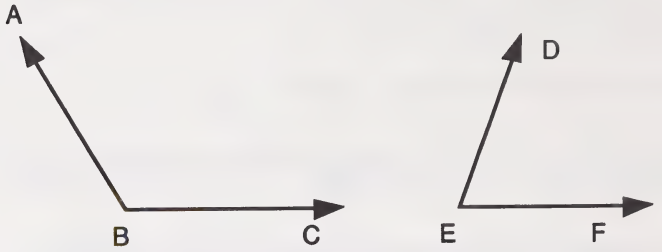
Estimating Angles

In order to estimate the measurement of angles, it is important to recognize 90°and 180° angles.



Example

Estimate the angles.



Solution

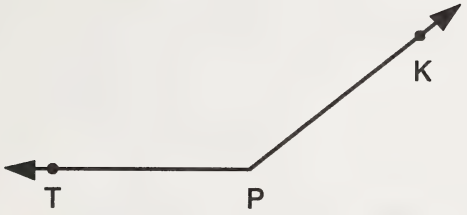
∠ABC is between 90° and 180°. It is about 135°.

∠DEF is less than a right angle. It is about 75°.

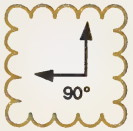
Using a Protractor

Example

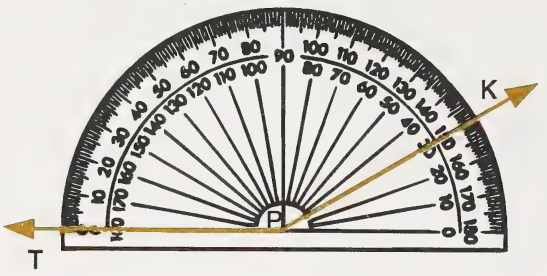
What is the measurement of ∠KPT?



First decide whether the angle is greater than or less than 90°.



Place your protractor on the angle. Be sure that the base line of the protractor is on one arm of the angle and that the centre of the protractor is at the vertex of the angle.



Decide whether to read the inner or outer scale.

Is the correct measurement 150° or 30°?

∠KPT = 150°

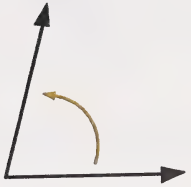
Introductory Activities

Space for Your Work

1. Measure the following angles with a protractor.

a.

b.



c.

d.

2. What do all the angles in Question 1 have in common?

3.

Space for Your Work

a.

b.

c.

d.

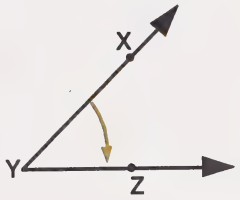
4. What do all the angles in Question 3 have in common?

See your learning facilitator to check your answers and to receive further instructions.

Working Together

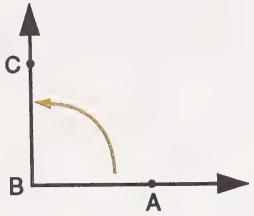
From the Introductory Activities you discovered that angles can be grouped according to their size.

If the measure of an angle is less than a quarter turn, the angle is called an **acute angle**.



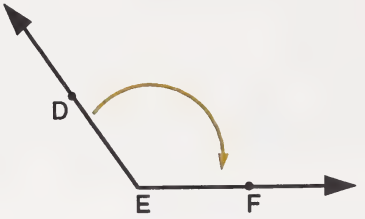
∠XYZ is an acute angle. It measures 45°.

If the measure of an angle is exactly a quarter turn, the angle is called a **right angle**.



∠ABC measures 90° and is a right angle.

If the measure of an angle is more than a quarter turn and less than a half turn, it is called an **obtuse angle**.

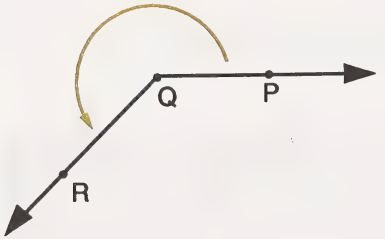


∠DEF is an obtuse angle. It measures 125°.

If the measure of an angle is exactly a half turn, the angle is called a **straight angle**.

∠MNO measures 180° and is a straight angle.

If the measure of an angle is more than half a turn and less than a full turn, it is called a **reflex angle**.



∠PQR is a reflex angle. It measures 135°.

Angles of Figures

Angles are formed where the line segments of a figure meet.

Example

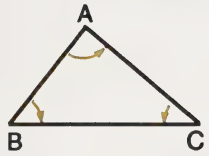


Figure ABC has three interior angles.

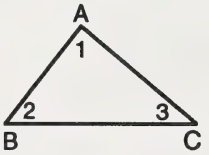
There is an angle where $\overline{BA}$ and $\overline{AC}$ meet. This angle may be called $\angle BAC$ or $\angle CAB$. The angle may also simply be called $\angle A$ since it is clear which angle is being named.

There is another angle where $\overline{BC}$ and $\overline{AC}$ meet. This angle may be called $\angle ACB$ or $\angle BCA$, or simply $\angle C$.

There is also an angle where $\overline{AB}$ and $\overline{BC}$ meet. This angle may be called $\angle ABC$ or $\angle CBA$, or simply $\angle B$.

Sometimes numbers are used to indicate the interior angles of a figure.

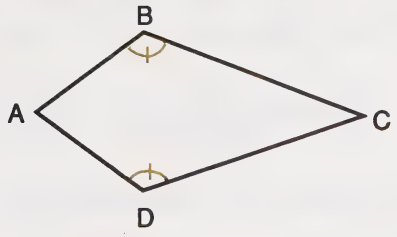
Example



The three angles of figure ABC may be referred to $\angle 1$, $\angle 2$, and $\angle 3$.

Congruent Angles

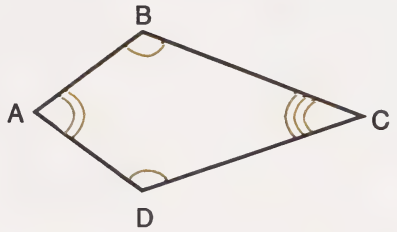
Congruent angles in a polygon may be indicated with an arc and slash marks.



$\angle B \cong \angle D$

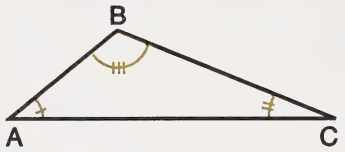
≅ means *is congruent to.*

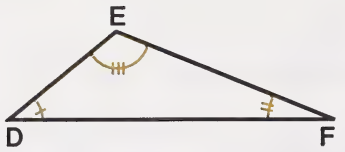
The congruent angles may also be indicated with the same number of arcs.



$\angle B \cong \angle D$

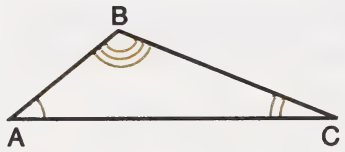
Congruent angles in two polygons may be indicated with an arc and slash marks.

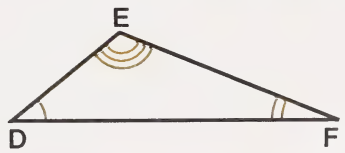




$\angle A \cong \angle D, \angle C \cong \angle F, \angle B \cong \angle E$

The congruent angles may also be indicated with the same number of arcs.





$\angle A \cong \angle D, \angle C \cong \angle F, \angle B \cong \angle E$

Practice Activities

Space for Your Work

1. For each angle, tell whether it is a right angle, an acute angle, an obtuse angle, or a straight angle.

a. 

b. 

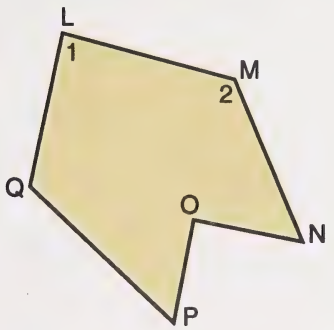
c. 

d. 

e. 

2. Give the measurements of the following angles in figure ∠LMNOPQ.

Space for Your Work



a. ∠1

b. ∠Q

c. ∠2

d. ∠QPO

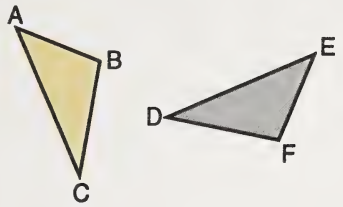
e. ∠PON

f. ∠N

3. Complete the blanks at the right. Use arcs and slash marks to indicate congruent angles in the figures.

Space for Your Work

a.

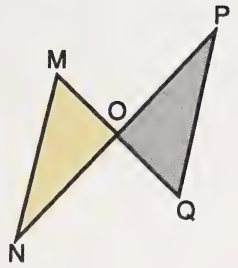


3. a. $\angle A \cong$ _____

$\angle B \cong$ _____

$\angle C \cong$ _____

b.



b. $\angle M \cong$ _____

$\angle N \cong$ _____

$\angle MON \cong$ _____

See your learning facilitator to check your answers and to receive further instructions.

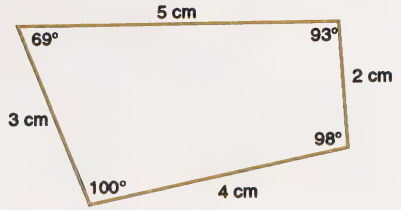
Concluding Activities

Space for Your Work

Print Alternative

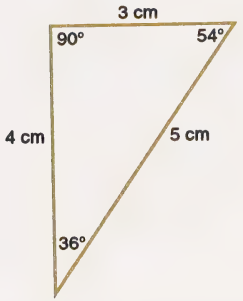
1. Use a protractor and ruler to make figures that are congruent to the following figures.

a.

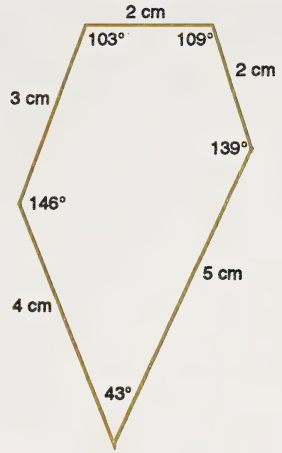


b.

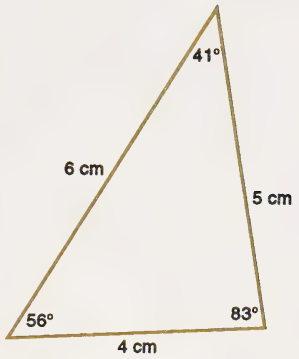
c.



d.



e.



Space for Your Work

Computer Alternative

Space for Your Work

2. Use LOGO to draw figures congruent to the ones in Question 1.

See your learning facilitator to check your answers and to receive further instructions.

What Lies Ahead

In this section you will learn these skills.

- classifying pairs of lines
- indicating parallel lines
- indicating perpendicular lines

In this section you will learn these terms.

- lines
- intersecting lines
- perpendicular lines
- parallel lines

Working Together

So far in this course you have classified curves into several categories.

- closed curves
- simple closed curves
- non-simple closed curves
- polygons
- convex polygons
- concave polygons

You have classified angles according to their size.

- acute angles
- right angles
- obtuse angles
- straight angles
- reflex angles

In this section you will classify lines.

Line

A line is a collection of points in a straight path that continues on forever in two directions.

Example

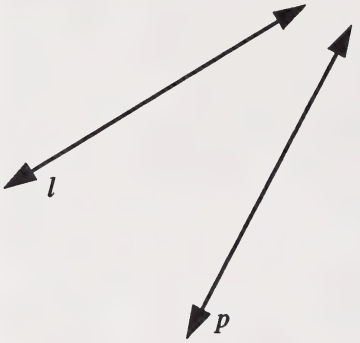
The arrowheads indicate that the line continues on forever.

You say *line MN* or *line NM.* The order does not matter.

You write $\overleftrightarrow{MN}$ or $\overleftrightarrow{NM}$.

When there are a number of lines, they are are sometimes labelled with a single lower case (small) letter.

Example

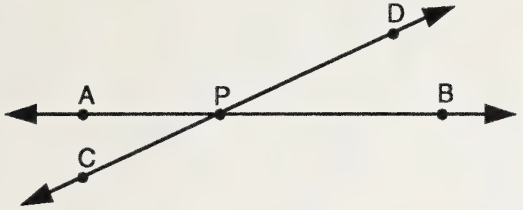


You say *line* l or *line* p.

You write l or p.

Sometimes lines cross or intersect and form angles.

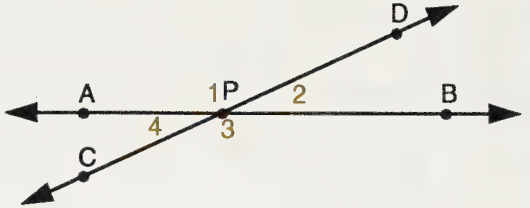
Examples



$\angle APD$, $\angle DPB$, $\angle BPC$, and $\angle APC$ are formed when $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ cross at P.

It is often easier to refer to the angles that are formed by using numbers.

Example



$\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are formed where $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ cross at P.

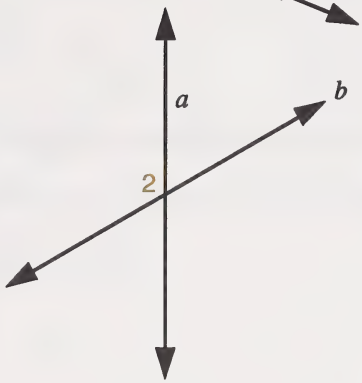
Introductory Activities

Space for Your Work

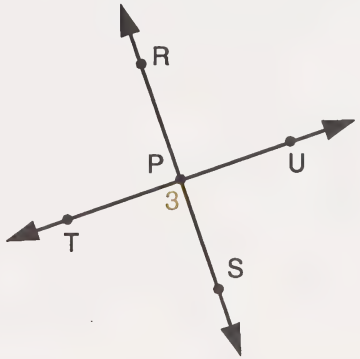
1. What are the measures of ∠1, ∠2, and ∠3 in the following questions?

a.

b.



c.



2. If you extended the following lines, do you think they will cross? Why or why not?

Space for Your Work

a.

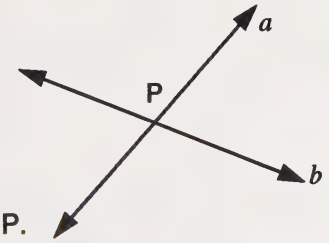
b.

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Intersecting Lines

Lines that cross or intersect are called **intersecting lines**.



Example

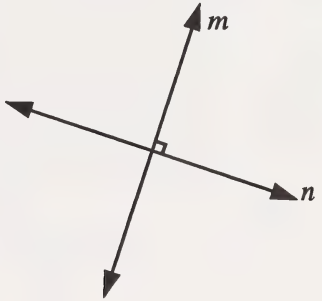
Line a crosses line b at P.

Perpendicular Lines

When lines meet to form a right angle, they are called **perpendicular lines**.

Example

The box indicates that the angle formed by two perpendicular lines is 90°.



Line m crosses line n at right angles.

Parallel Lines

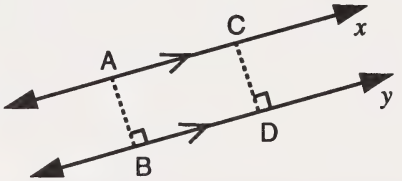
Lines that will never cross are called **parallel lines**.

Example

Lines x and y are parallel. The ➢ symbol on both lines indicates that the lines are parallel.

Note

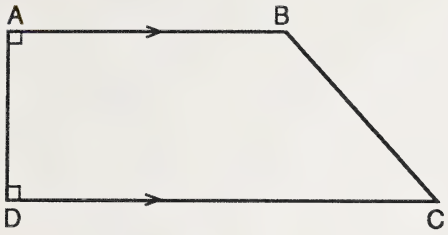
The perpendicular distance between parallel lines is the same anywhere on the lines.



$\overline{AB} \cong \overline{CD}$

Line segments and sides of figures are parts of lines. The sides of figures may be parallel or perpendicular.

Example



The ➢ symbol indicates that $\overline{AB}$ and $\overline{DC}$ are parallel lines.

$\overline{AB} \parallel \overline{DC}$

// means *is parallel to.*

The ⊾ symbol indicates that $\overline{AD}$ and $\overline{DC}$, and $\overline{AB}$ and $\overline{AD}$ are perpendicular lines.

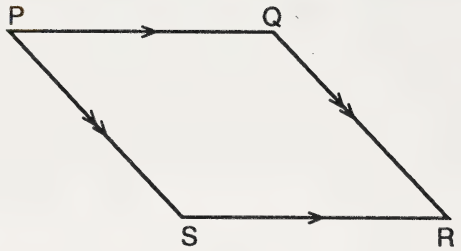
$\overline{AD} \perp \overline{DC}$

$\overline{AB} \perp \overline{AD}$

⊥ means *is perpendicular to.*

Figures may have more than one pair of parallel lines.

Example



$\overline{PQ} \parallel \overline{SR}$

$\overline{PS} \parallel \overline{QR}$

Notice that the parallel lines have matching symbols.

Practice Activities

Space for Your Work

1. Which of the following letters contain parallel lines?

   a. E

   b. V

   c. N

   d. Z

   e. H

2. Which of the following letters contain perpendicular lines?

Space for Your Work

a. E

b. V

c. N

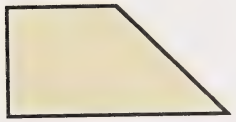
d. Z

e. H

3. Which of the following figures have parallel sides? Label the sides that are parallel.

Space for Your Work

a. 

b. 

c. 

d. 

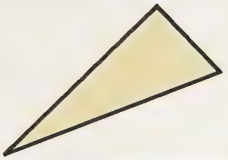
4. Which of the following figures have perpendicular sides? Label the sides that are perpendicular.

Space for Your Work

a. 

b. 

c. 

d. 

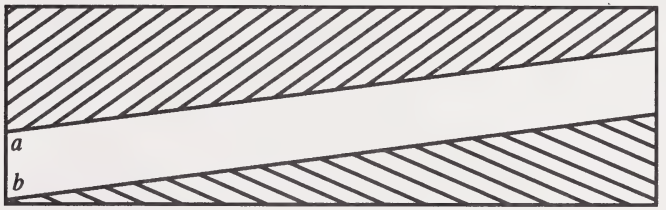
See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

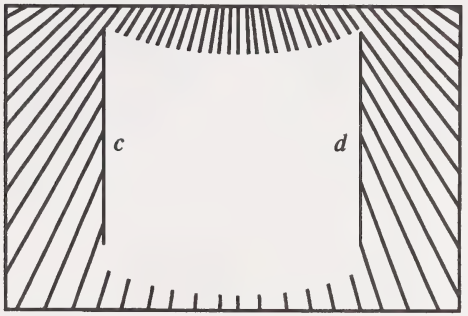
Space for Your Work

1. Things are not always what they appear to be.

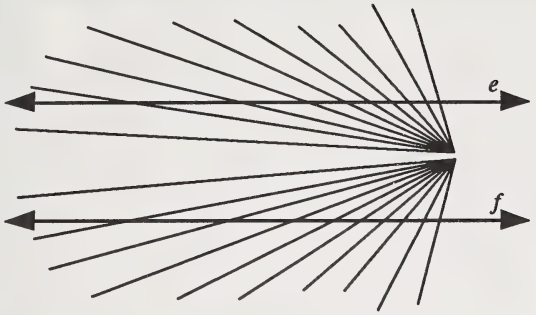
   a. Are lines a and b parallel?



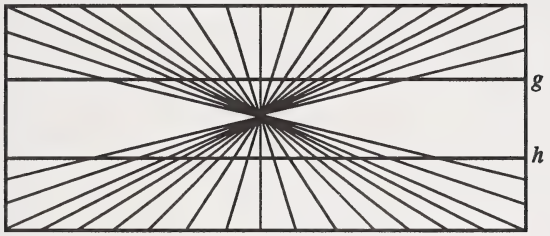
   b. Are lines c and d parallel?



c. Are lines e and f parallel?



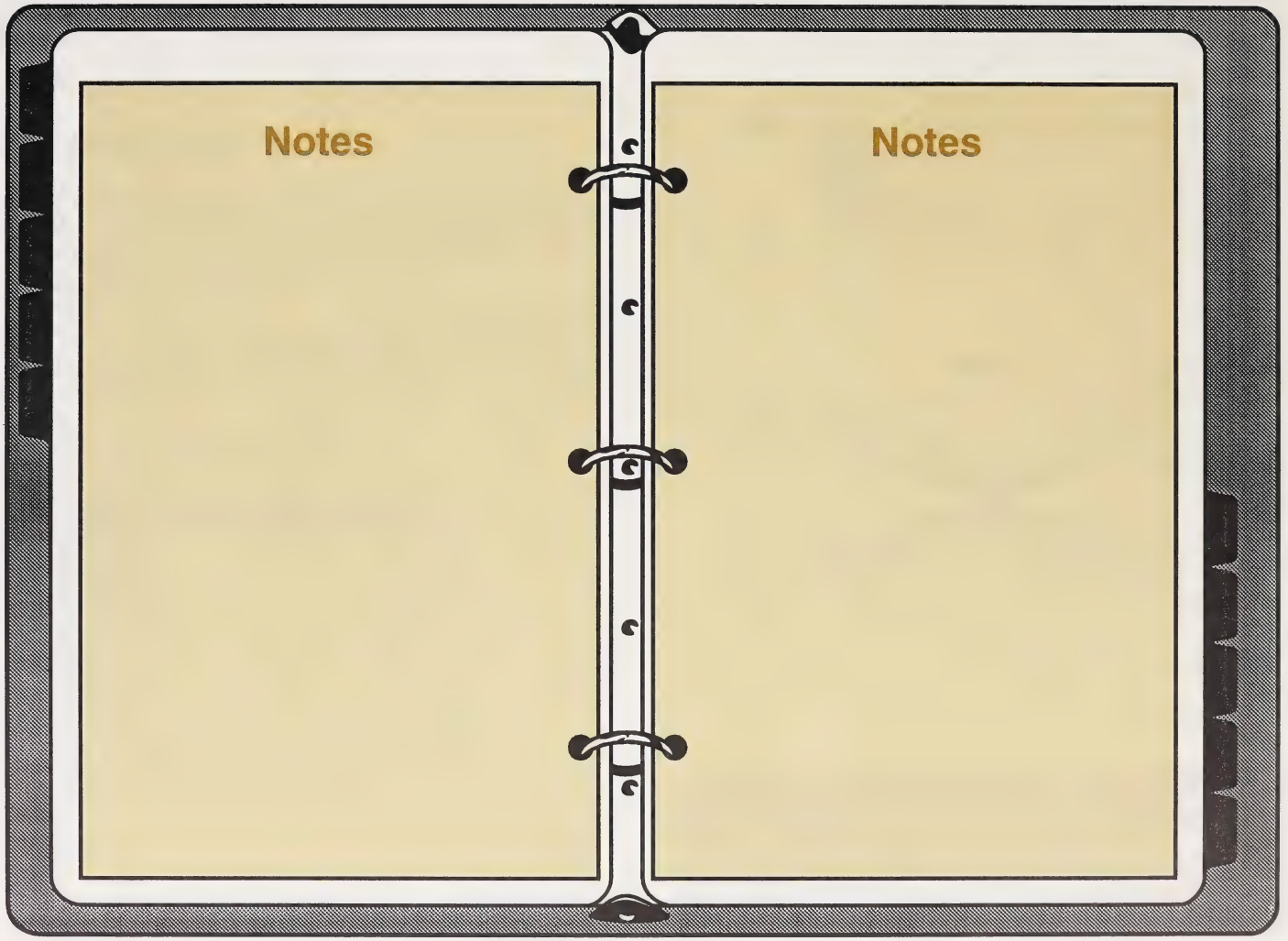
d. Are lines g and h parallel?



2. Create your own optical illusion.

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.


Notes
Notes

What Lies Ahead

In this section you will classify polygons according to the number of sides and angles.

In this section you will learn these terms.

- triangle
- quadrilateral
- pentagon
- hexagon
- heptagon
- octagon
- nonagon
- decagon
- adjacent vertex
- non-adjacent vertex
- diagonal
- regular polygon

Working Together

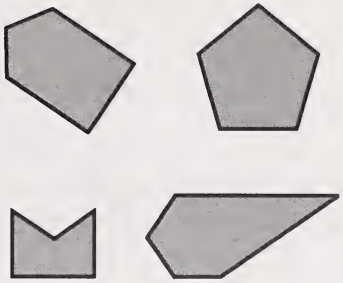
As you will recall, a polygon is a simple closed figure made up of line segments.

In this section you will classify polygons according to the characteristics or properties of their sides and angles.

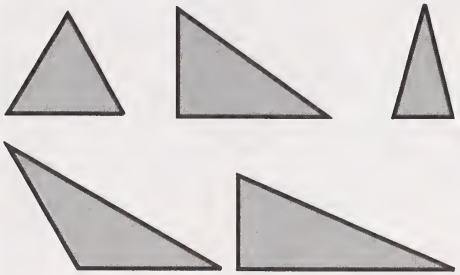
Introductory Activities

Space for Your Work

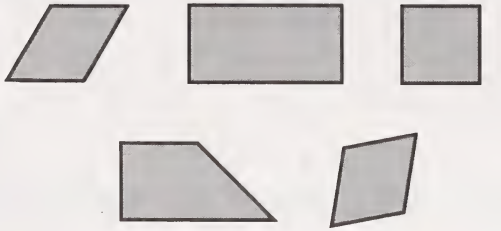
1. What characteristics do these polygons share?



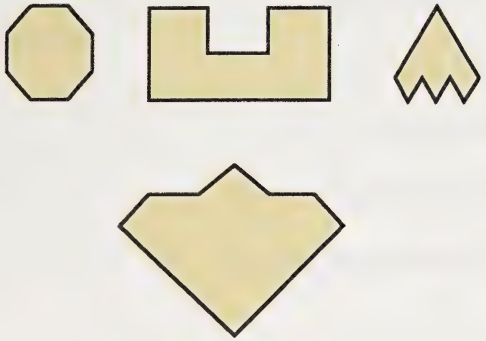
2. What characteristics do these polygons share?



3. What characteristics do these polygons share?



4. What characteristics do these polygons share?



Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Polygons can be classified according to how many sides they have.

Polygons have names related to the number of sides and angles.

Name of Polygon	Number of Sides	Number of Angles
triangle	3	3
quadrilateral	4	4
pentagon	5	5
hexagon	6	6
heptagon	7	7
octagon	8	8
nonagon	9	9
decagon	10	10

tri- means *three*

quad- means *four*

penta- means *five*

hexa- means *six*

hepta- means *seven*

octa- means *eight*

nona- means *nine*

deca- means *ten*

poly- means *many*

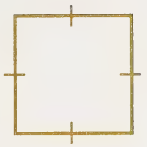
Notice that most of the names end in *-gon.* The ending *-gon* means *angle.*

Notice that the number of sides and the number of angles in a polygon are the same.

Polygons that have all congruent sides and all congruent angles are called **regular polygons**.

Examples

regular triangle

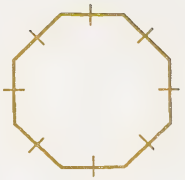


regular quadrilateral

regular pentagon

regular hexagon

regular heptagon



regular octagon

Practice Activities

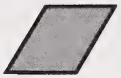
Space for Your Work

1. Match the following figures with their names.

a. 

b. 

c. 

d. 

e. 

f. 

g. 

h. 

___ triangle

___ quadrilateral

___ pentagon

___ hexagon

___ octagon

___ nonagon

___ decagon

2. Which of the figures in Question 1 are regular polygons?

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

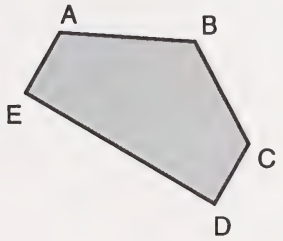
Space for Your Work

Computer Alternative

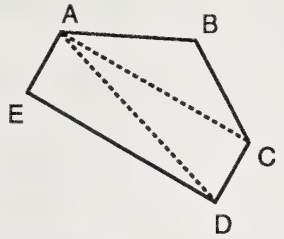
1. Do the program "Diagonals" in *Problem Solving Strategies* (MECC).

Print Alternative

2. The vertices of a polygon are refered to as **adjacent vertices** and **non-adjacent vertices**. Adjacent vertices are next to each other. Non-adjacent vertices are not next to each other. Name the vertices that are adjacent to A.



3. A **diagonal** is a line segment joining two non-adjacent vertices in a polygon. For example, in figure ABCDE, there are two diagonals from A.



a. How many diagnonals are there from B?
b. How many diagonals are there in polygon ABCDE in total? Do not count the same segment twice.

4. a. Complete the following table. Use the polygons in the Appendix to help you.

Kind of Polygon	Number of Sides	Number of Diagonals
triangle		
quadrilateral		
pentagon		
hexagon		

b. Can you see a pattern in part a.? Explain the pattern.
c. Use the pattern to find the number of diagonals in a decagon. Do not use a drawing or count the diagonals.

Space for Your Work

5. Use all the seven tangram pieces in the Appendix to form the following polygons.

   a. a pentagon

   b. a hexagon

Space for Your Work

6. A polygon with twelve sides and angles is called a **dodecagon**. Use all six pieces of the regular dodecagon in the Appendix to form a square.

See your learning facilitator to check your answers and to receive further instructions.

What Lies Ahead

In this section you will learn this skill.

- classifying triangles

In this section you will use these words.

- equilateral triangle
- equiangular triangle
- isosceles triangle
- scalene triangle
- acute triangle
- obtuse triangle
- right triangle

Working Together

In this section you will examine a number of different triangles to see if you can discover some of their common characteristics.

Introductory Activities

Space for Your Work

Questions 1 to 4 require you to use the triangles from the Appendix.

1. Use your protractor to carefully measure the angles in each of the triangles. Write the meaure inside each angle.

2. Measure the lengths of the sides of the triangles. Write the measures along the inside of the triangles.

3. Cut out each triangle and fold it (or use a MIRA) to determine the number of lines of symmetry. Put this number in the inside of the triangle.

4. Sort the triangles into groups using the following criteria and record the triangles in each group.

   a. classification of largest angle (acute, right, obtuse)

   b. number of equal sides (0, 2, 3)

   c. number of lines of symmetry (0, 1, 3)

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

Working Together

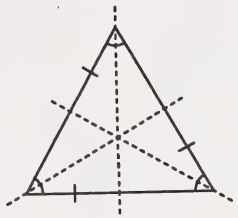
Classifying Triangles

From your investigations in the Introductory Activities, you discovered common characteristics of triangles.

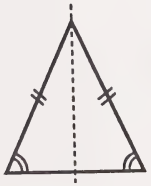
Triangles are classified and named in two ways:

- by the number of congruent parts
- by the measure of their largest angle

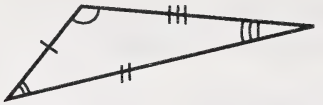
Congruent Parts



Equilateral triangles or **equiangular triangles** have three congruent sides and three congruent angles. Three lines of symmetry divide these triangles into three symmetrical parts.



Isosceles triangles have two congruent sides and two congruent angles. One line of symmetry divides these triangles into two symmetrical parts.

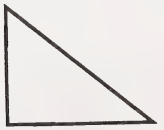


Scalene triangles have no congruent sides and no congruent angles. There are no lines of symmetry, so there are no symmetrical parts in these triangles.

Largest Angles

The largest angle of an **acute triangle** is less than 90°.

The largest angle of an **obtuse triangle** is between 90° and 180°.



The largest angle of a **right triangle** is equal to 90°.

Practice Activities

Space for Your Work

1. Match the type of triangle with its description.

a.	equiangular	___	two equal sides
b.	right-angled	___	three equal angles
c.	acute-angled	___	three equal sides
d.	obtuse-angled	___	one obtuse angle
e.	equilateral	___	no equal sides
f.	isosceles	___	three acute angles
g.	scalene	___	one 90° angle

2. Six triangles have the following measurements. Classify each triangle in two ways.

   a. 40°, 60°, 80°

   b. 60°, 60°, 60°

   c. 45°, 90°, 45°

   d. 120°, 40°, 20°

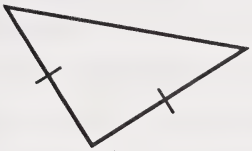
   e. 40°, 40°, 100°

   f. 30°, 60°, 90°

3. Classify each of the triangles in two ways.

Space for Your Work

a. 

b. 

c. 

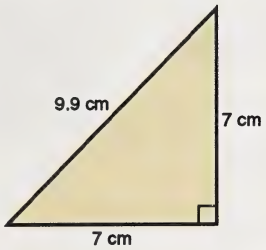
See your learning facilitator to check your answers and to receive further instructions.

Extra Practice

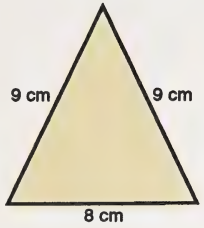
Space for Your Work

Classify each triangle two ways.

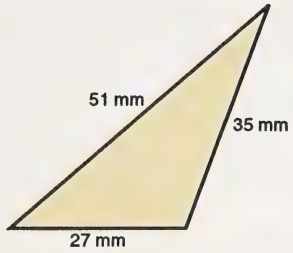
1.



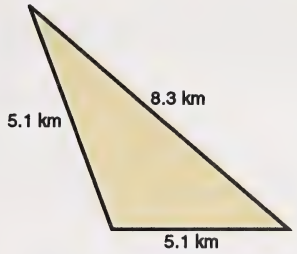
2.



3.

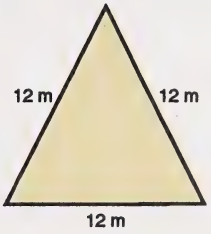


4.

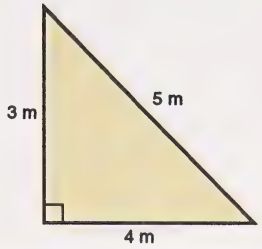


Space for Your Work

5.



6.



See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

Space for Your Work

Print Alternative

1. Can a triangle be both of the following? Show why or why not.

   a. scalene and obtuse

   b. equilateral and obtuse

   c. scalene and right

   d. isosceles and right

   e. isosceles and equiangular

   f. equilateral and acute

   g. scalene and isosceles

2. Can a triangle have these characteristics? Show why or why not.

   a. two right angles

   b. two obtuse angles

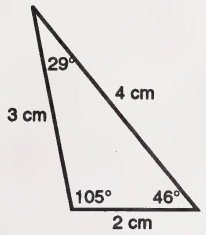
   c. exactly two lines of symmetry

3. For each triangle, do the following.

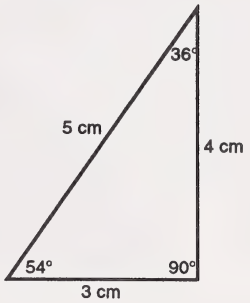
Space for Your Work

- List the angle measures in order of size from smallest to largest.
- List the lengths of the sides in order from smallest to largest.

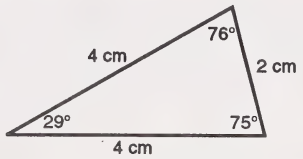
a.



b.



c.



4. Did you discover a pattern in Question 3? What seems to be the relationship between the measure of each angle and the length of the side opposite the angle?

5. For each triangle in Question 4, list the measure of the longest side and the sum of the measures of the other two sides.

6. What pattern did you notice in Question 5? Do you think a triangle can have sides that measure 3 cm, 4 cm, and 7 cm? Show why or why not.

7. Use a protractor and a ruler to draw each of the following figures.

   a. an isosceles triangle with two 70° angles

   b. a right triangle with the two smaller sides measuring 4 cm and 6 cm

Space for Your Work

Computer Alternative

Space for Your Work

8. Draw the triangles described in Question 7 on a computer using LOGO.

See your learning facilitator to check your answers and to receive further instructions.

What Lies Ahead

In this section you will learn this skill.

- classifying quadrilaterals

In this section you will use these words.

- trapezoid
- parallelogram
- rectangle
- rhombus
- square
- kite

Working Together

This section deals with quadrilaterals (four-sided polygons). You will examine a number of quadrilaterals to see what characteristics they have in common. You will then use these characteristics to see how various kinds of quadrilaterals are related.

Introductory Activities

Space for Your Work

Questions 1 to 3 require you to use the quadrilaterals from the Appendix.

1. a. Do the following for each of the quadrilaterals.
   - Measure the sides and write the measures inside the figure.
   - Indicate the congruent sides.
   - Measure the angles and indicate their measures in the figure.
   - Indicate the right angles with the symbol ⌐.
   - Identify the parallel sides and indicate them with the symbols ➢ or ➢➢.

   b. Identify the quadrilaterals that have at least one pair of parallel sides.

   c. Identify the quadrilaterals that have two pairs of parallel sides.

   d. Identify the quadrilaterals that have congruent opposite sides.

   e. Identify the quadrilaterals that have four congruent sides.

   f. Identify those figures that have four right angles.

2. Cut out each figure that you examined in Question 1 and fold it (or use a MIRA) to determine the number of lines of symmetry. Put this number inside each quadrilateral.

3. Which figures have the following characteristics?

   a. no lines of symmetry

   b. one line of symmetry

   c. two lines of symmetry

   d. four lines of symmetry

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

Working Together

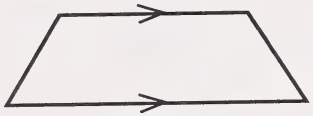
Classifying Quadrilaterals

From your investigations in the Introductory Activities, you discovered common characteristics of quadrilaterals.

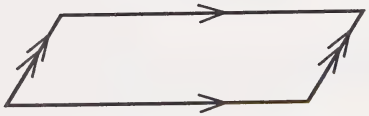
Quadrilaterals may be classified in five ways:

- by the number of parallel sides
- by the presence of four congruent sides
- by the presence of four right angles
- by the presence of four right angles and four congruent sides
- by the number of lines of symmetry

Parallel Sides



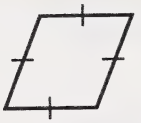
A **trapezoid** has at least one pair of parallel sides.



A **parallelogram** has two pairs of parallel sides.

A parallelogram is a special type of trapezoid.

Four Congruent Sides



A **rhombus** has four congruent sides.

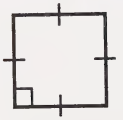
A rhombus is a special type of parallelogram.

Four Right Angles

A **rectangle** has four right angles.

A rectangle is a special type of parallelogram.

Four Right Angles and Four Congruent Sides

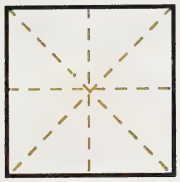


A square has four right angles and four sides the same length.

A square is a special type of rectangle.

Lines of Symmetry

A **square** has four lines of symmetry.

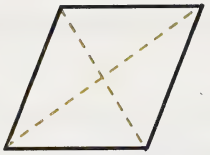


A square has four congruent sides and four congruent angles.

A **rectangle** has two lines of symmetry.

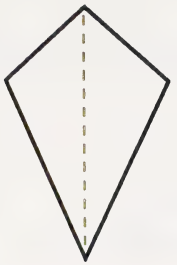
A rectangle has congruent opposite sides and four pairs of congruent angles.

A **rhombus** has two lines of symmetry.



A rhombus has four congruent sides and congruent opposite angles.

A **kite** also has one line of symmetry.



A kite has congruent adjacent sides and congruent opposite angles.

All the other quadrilaterals have no lines of symmetry.

Relating Quadrilaterals

By now you should be aware that quadrilaterals may have more than one name.

The diagram at the right shows how quadrilaterals may be organized. Every figure on the diagram may be classified by any name above it to which it is connected.

Example 1

A rectangle is also a parallelogram, a trapezoid, and a quadrilateral.

Example 2

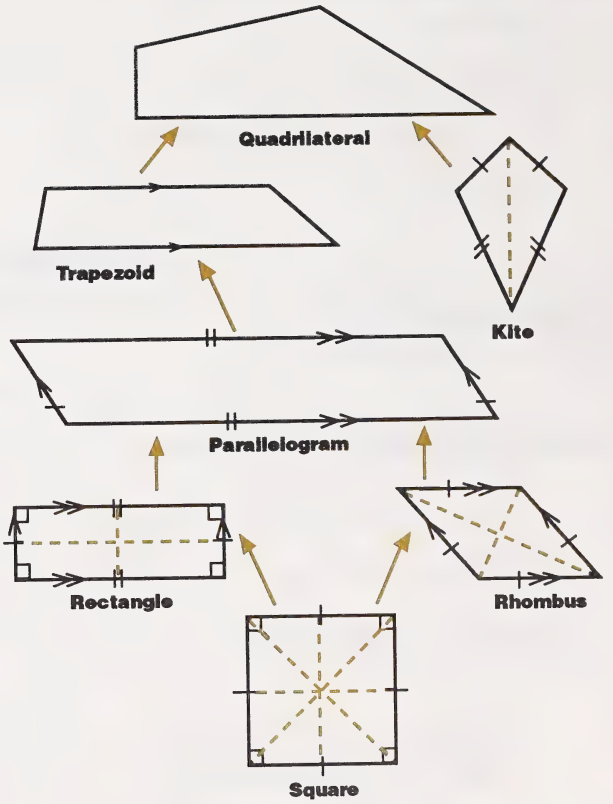
A rhombus is also a parallelogram, a trapezoid, and a quadrilateral.

Example 3

A square is also a rectangle, a rhombus, a parallelogram, a trapezoid, and a quadrilateral.

Note

The most specific name for a figure is the one furthest down the diagram.

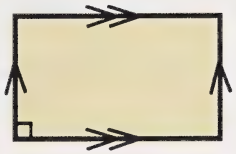


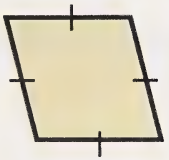
Practice Activities

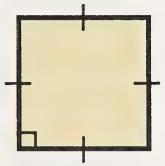
Space for Your Work

1. Give two names that could be used for each of the following figures. Underline the one that best describes the figure.

a. 

b. 

c. 

d. 

2. a. How is a square different from a rhombus?

   b. How are they similar?

Space for Your Work

3. Give the number of lines of symmetry for each figure.

   a. trapezoid

   b. parallelogram

   c. rectangle

   d. rhombus

   e. square

   f. kite

4. Answer **yes** or **no** to these questions.

   a. Is every rhombus a parallelogram?

   b. Is every square a rectangle?

   c. Is every parallelogram a rectangle?

   d. Is every rectangle a parallelogram?

   e. Is every square a rhombus?

   f. Is every rhombus a square?

   g. Is every rectangle a square?

   h. Is every trapezoid a parallelogram?

5. Which quadrilaterals have congruent opposite angles and congruent opposite sides?

Space for Your Work

6. Draw (if possible) the quadrilaterals that have these characteristics.

   a. at least one right angle and four congruent sides

   b. exactly two right angles and no parallel sides

   c. exactly one right angle and one pair of parallel sides

7. Name the quadrilaterals that you drew in Question 6.

See your learning facilitator to check your answers and to receive further instructions.

Extra Practice

Space for Your Work

Complete the puzzle on the following page.[1]

See your learning facilitator to check your answers and to receive further instructions.

[1] Creative Publications for excerpt from *MIDDLE SCHOOL MATH WITH PIZZAZZ! BOOK D* ©1989, Sunnyvale, California 94086

Why Didn't the Snobbish Potatoes Want Their Daughter to Marry a News Broadcaster?

Under each figure, circle the number-letter combination next to each word that correctly names the figure. Write the letter in the matching numbered box at the bottom of the page.

1

5-A parallelogram
16-0 rectangle
19-F square

2

25-E parallelogram
13-I rectangle
4-D rhombus

3

9-U quadrilateral
21-F parallelogram
1-H trapezoid

4

20-N parallelogram
11-T rectangle
23-A square

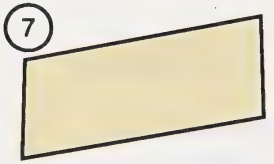
5

2-E quadrilateral
24-V parallelogram
8-P rhombus

6

19-O quadrilateral
15-L rectangle
6-S rhombus

7

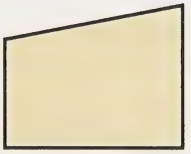


13-A quadrilateral
26-R parallelogram
7-N trapezoid

8

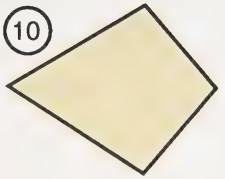
17-M rectangle
10-P square
14-S trapezoid

9



21-E parallelogram
18-I rhombus
8-J trapezoid

10



4-W quadrilateral
12-O parallelogram
24-N trapezoid

11

22-T quadrilateral
15-C rhombus
3-B square

12

10-S rectangle
18-M rhombus
24-T square

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

Concluding Activities

Space for Your Work

Computer Alternative

1. Draw different kinds of quadrilaterals on the computer using LOGO.

Print Alternative

Questions 2 to 5 require you to use the quadrilaterals from the Appendix.

2. For each figure, do the following.
   - Draw all the diagonals.
   - Measure the lengths of the diagonals and write the measures inside the figure.
   - Measure the angles at which the diagonals cross and indicate the right angles with the symbol ⌐ .

3. a. Which figures have congruent diagonals?

   b. What kind of quadrilaterals are these?

4. a. Which figures have diagonals that meet at right angles?

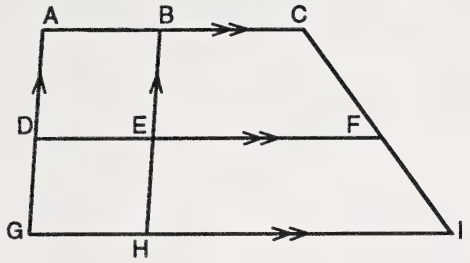
   b. What kind of quadrilaterals are these?

5. a. Which figures have congruent diagonals that meet at right angles?

   b. What kind of quadrilaterals are these?

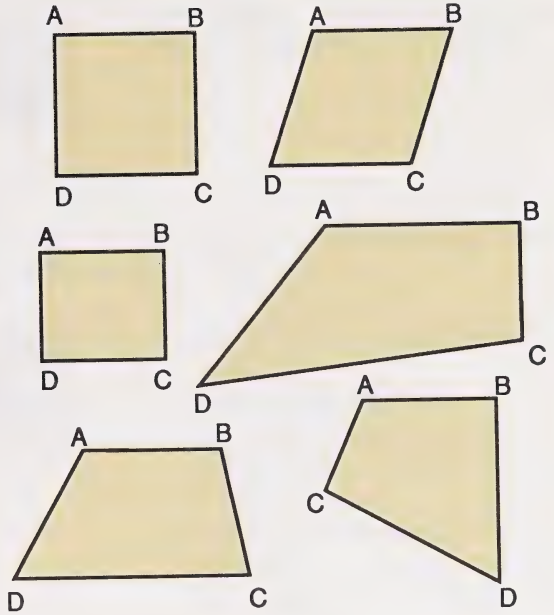
Space for Your Work

6. Name three parallelograms and four trapezoids in this diagram.



7. Use a protractor to bisect each angle of this parallelogram. Extend the lines that bisect each angle until they intersect one another. What figure is created by the intersection of these lines? Hint: Bisect means to divide into two equal parts.

8. Find the midpoints of each side of these figures. Join the midpoints together to form another four-sided figure. What do you notice about the opposite sides of these new four-sided figures?



9. Use all seven tangram pieces from the Appendix to form a rectangle, a square, and a parallelogram.

See your learning facilitator to check your answers and to receive further instructions.

Space for Your Work

What Lies Ahead

In this section you will learn these terms.

- right prism
- right rectangular prism
- cube
- face
- edge
- vertex
- base
- lateral face

Working Together

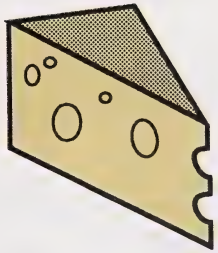
So far in this module you have been investigating two-dimensional figures. **Two-dimensional figures** have length and width.

In this section you will explore three-dimensional objects. **Three-dimensional objects** have length, width, and height.

Introductory Activities

Space for Your Work

1. Examine a wedge of cheese.



   a. How many flat surfaces (faces) does the wedge have?

   b. What shape is each flat surface?

   c. Which faces are congruent?

2. Examine an unsharpened pencil.

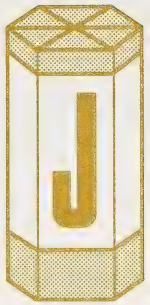
   a. How many flat surfaces (faces) does the pencil have?

   b. What shape is each flat surface?

   c. Which faces are congruent?

3. Examine a box of disposable cloths.

Space for Your Work



a. How many flat surfaces does the box have?

b. What shape is each flat surface?

c. Which faces are congruent?

4. Examine a cereal box, toothpaste box, or tissue box.

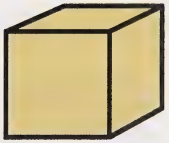
a. How many flat surfaces (faces) does the box have?

b. What shape is each flat surface (face) of the box?

c. Which faces are congruent?

5. Examine a sugar cube or child's building block.

Space for Your Work



a. How many flat surfaces (faces) does it have?

b. What shape is each surface (face)?

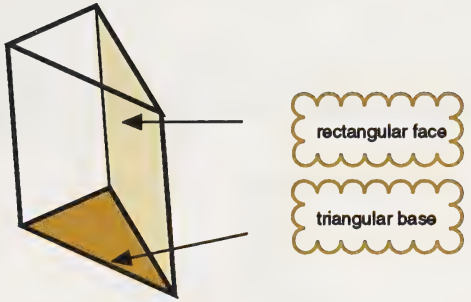
c. Which faces are congruent?

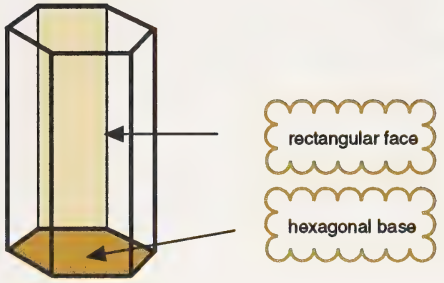
See your learning facilitator to check your answers and to receive further instructions.

Working Together

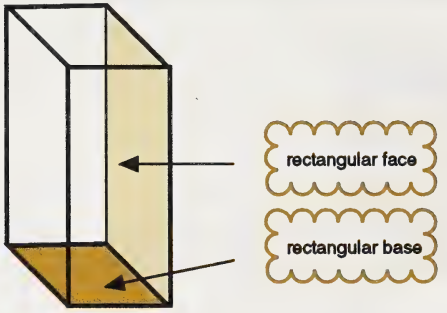
A **right prism** has two congruent **bases**. The remaining flat surfaces are rectangles.

Examples



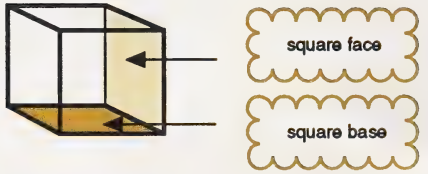


A **right rectangular prism** has rectangular bases.

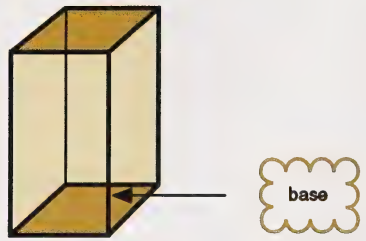


A **cube** is a special right rectangular prism. All the faces are squares.

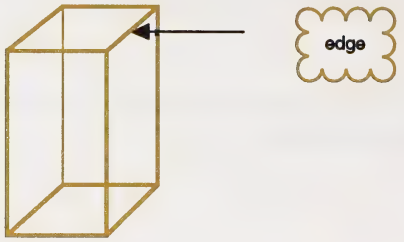
Example



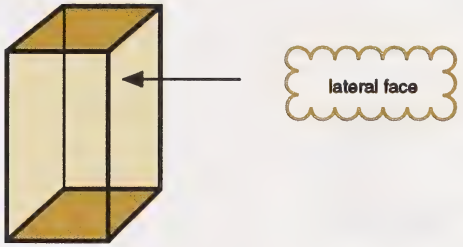
The top and bottom faces of a right rectangular prism are called the **bases**. The bases are congruent.



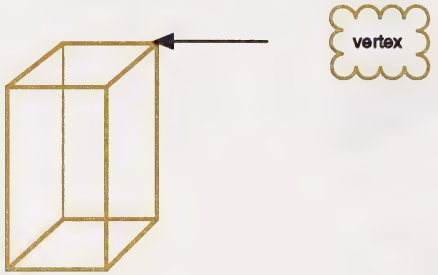
Where two faces meet is called an **edge**.



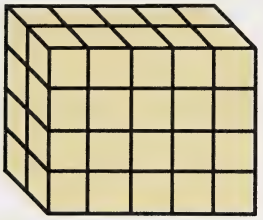
The other faces of a right rectangular prism are called **lateral faces**.



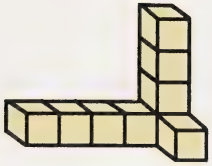
Where two edges meet is called a **vertex**.



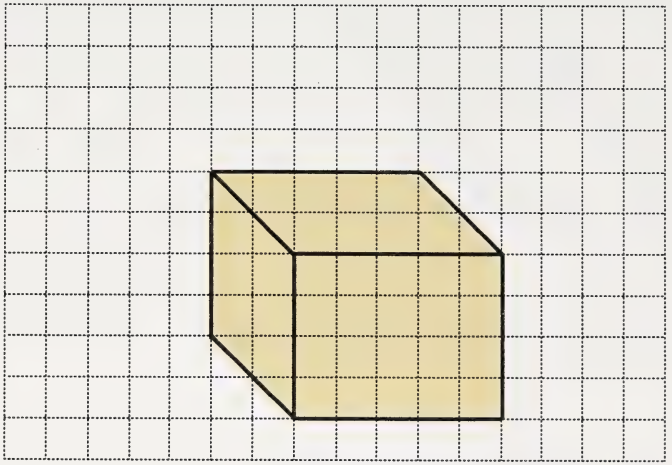
You can make right rectangular prism using blocks.

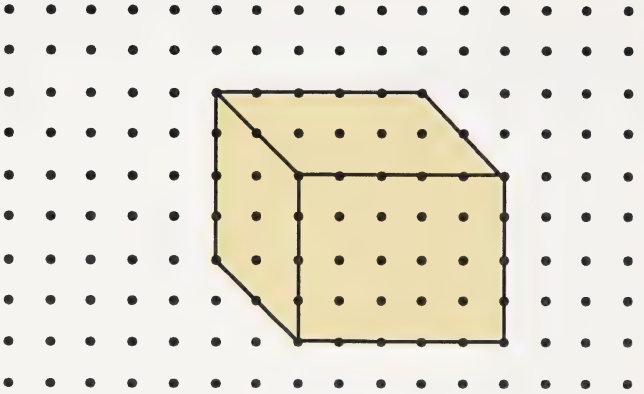


The base of this right rectangular prism is five blocks long and two blocks wide. The height of the rectangular right prism is four blocks.



You can sketch a right rectangular prism using graph paper or dot paper.





Practice Activities

Space for Your Work

1. Examine a cereal box, toothpaste box, or tissue box.

   a. How many edges does a right rectangular prism have?

   b. Are any pairs of edges perpendicular?

   c. Are any pairs of edges parallel?

   d. How many vertices are there?

   e. Are opposite faces congruent?

2. Use blocks to build right rectangular prisms with the following dimensions. Then draw sketches of the right rectangular prisms on the dot paper provided in the Appendix.

   a. four blocks long, three blocks wide, two blocks high

   b. five blocks long, four blocks wide, three blocks high

   c. five blocks longs, two blocks wide, three blocks high

   d. three blocks long, three blocks wide, three blocks high

3. Which of the right rectangular prisms in Question 2 is a cube?

See your learning facilitator to check your answers and to receive further instructions.

Space for Your Work

Concluding Activities

Space for Your Work

Here are four views of the same cube. Which designs are opposite each other on the cube?

See your learning facilitator to check your answers and to receive further instructions.

What Lies Ahead

In this section you will review these concepts.

- naming points, line segments, angles, and polygons
- interpreting simple closed curves
- interpreting polygons
- interpreting convex and concave polygons
- classifying angles
- classifying lines
- classifying figures according to number of sides and angles
- classifying triangles
- classifying quadrilaterals
- interpreting right rectangular prisms and cubes

Working Together

At this point, it may be a good idea to review the skills you have learned in Part Two.

Turn to Section 2 and review the Pretest. Then correct any errors you may have made at the time. You may be pleasantly surprised to discover how much you have learned!

WESTFILE INC.

Sections 11 to 21 deal with measurement.

In Grade 7 you learned to measure perimeter, area, and volume directly.

In Grade 8, you will use formulas to calculate perimeter, area, and volume indirectly.

What Lies Ahead

In this section you will review these skills.

- estimating and measuring length, mass, capacity, perimeter, area, and volume
- comparing the areas of figures with the same perimeter
- comparing the perimeters of figures with the same area
- relating volume and capacity in the metric system
- changing from one unit to another

Working Together

To begin Part Two, you will examine the measurement skills with that you developed in previous schooling. It is important that you maintain these skills.

The following Pretest will help you and your learning facilitator discover your strengths and weaknesses.

Review

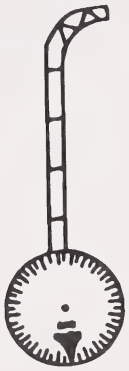
Space for Your Work

1. Define *measurement.*

2. Why do you think the metric system is used by most of the countries in the world?

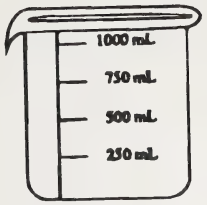
3. Can you ever measure absolutely accurately? Why or why not?

4. What do the following instruments measure?

   a. 

   b. 

Space for Your Work

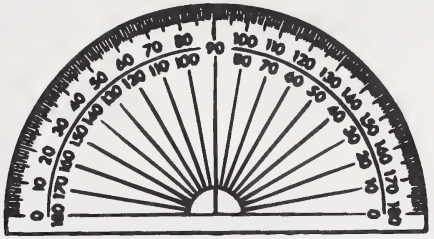
c. 

d. 

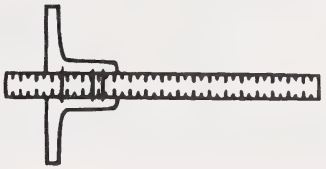
e. 

Space for Your Work

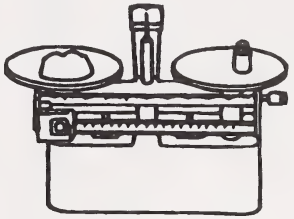
f.



g.



h.

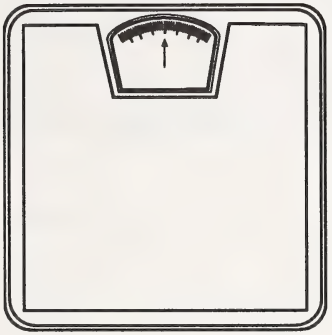


i.

j. 

Space for Your Work

k. 

l. 

5. What unit would be appropriate to measure each of these quantities?

Space for Your Work

   a. the distance from Calgary to Banff

   b. the depth of the sea

   c. the length of a fire hose

   d. the width of a book

   e. the thickness of a sheet of paper

   f. your height

6. Is each statement reasonable? Answer **yes** or **no**.

   a. The pencil is 7 cm long.

   b. The mosquito is 7 m long.

   c. The flagpole is 7 mm long.

   d. The bike trail is 7 km long.

7. Measure the following line segments.

Space for Your Work

a. _____

b. _____

c. _____

8. What unit would you use to measure each of these masses?

a. a stove

b. a toaster

c. a box of paper clips

d. yourself

e. a hair

9. Is each statement reasonable? Answer **yes** or **no**.

Space for Your Work

a. A motorcycle has a mass of 0.3 t.

b. A tennis ball has a mass of 3 kg.

c. A concrete block has a mass of 11 kg.

d. A bicycle has a mass of 11 g.

e. A basketball has a mass of 566 g.

f. A bag of potato chips has a mass of 450 g.

10. What unit would you use to measure the capacity of each of these items?

a. a tube of toothpaste

b. a carton of milk

c. a tanker truck

d. a bottle of pop

e. a honey jar

f. an eye dropper

11. Is each statement reasonable? Answer **yes** or **no**.

Space for Your Work

a. A hot-water tank has a capacity of 180 mL.

b. A cereal bowl has a capacity of 225 mL.

c. A drinking straw has a capacity of 5 mL.

d. A water balloon has a capacity of 250 mL.

e. A garbage can has a capacity of 15 L.

f. A bottle cap has a capacity of 1 L.

12. Which unit would you use to measure the area of each of the following items?

a. a garden

b. a place mat

c. a farm

d. a province

e. a stamp

13. Is each statement reasonable? Answer **yes** or **no**.

Space for Your Work

a. The area of a hockey rink is 1586 km^2.

b. The area of a credit card is 46.75 cm^2.

c. The area of a felt pennant is 0.3 m^2.

d. The area of a stop sign is 4320 cm^2.

e. The area of a ballpark is 5.1 ha.

14. What unit would you use to measure the volume of each of the following items?

a. a hamster cage

b. a moving truck

c. a box of cereal

d. a swimming pool

15. Is each statement reasonable? Answer **yes** or **no**.

a. The volume of a walnut is 12 m^3.

b. The volume of a washroom is 0.1 m^3.

c. The volume of a softball is 480 cm^3.

d. The volume of a loaf of bread is 3500 m^3.

16. Complete the following unit conversions.

Space for Your Work

a. 30 cm = ▩ mm

b. 152 mm = ▩ m

c. 3 L = ▩ mL

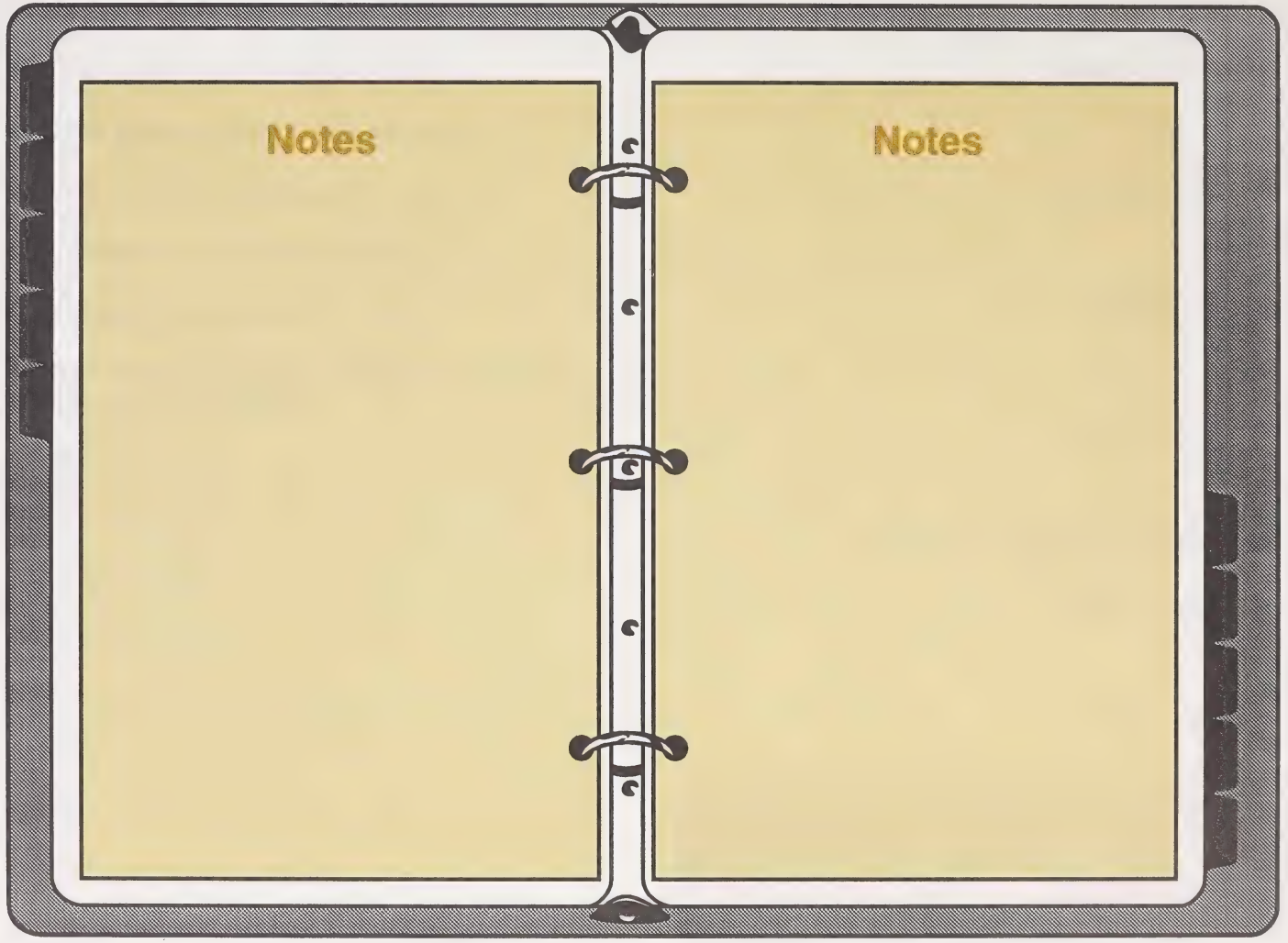
d. 518 g = ▩ kg

17. Complete the following conversions.

a. 13 mL = ▩ cm^3

b. 2 L = ▩ cm^3

See your learning facilitator to check your answers and to receive further instructions.


Notes
Notes

What Lies Ahead

In this section you will learn these skills.

- calculating the perimeter of rectangles, parallelograms, and regular polygons using formulas
- calculating the circumference of circles using a formula
- calculating the area of rectangles and squares, parallelograms, triangles, trapezoids, and circles using formulas
- calculating the volume of right rectangular prisms and cubes using formulas

Working Together

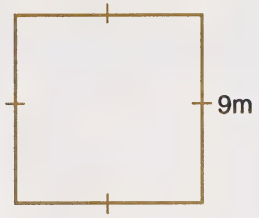
The Pretest in this section will help you and your learning facilitator discover your strengths and weaknesses in measurement.

Pretest

Space for Your Work

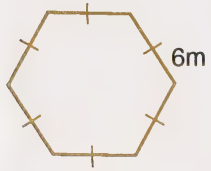
1. Write a formula and then find the perimeter of each polygon.

a.



b.

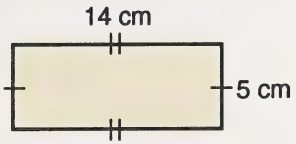
c.

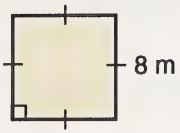


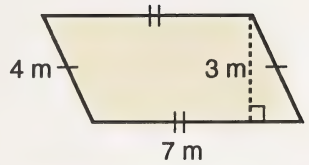
2. Write a formula and find the circumference of the circle.

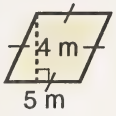
Space for Your Work

3. Write a formula and find the area of the following quadrilaterals.

a. 

b. 

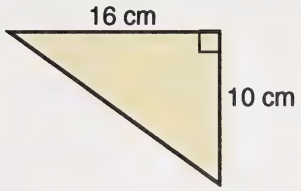
c. 

d. 

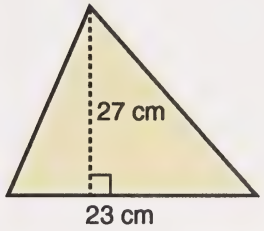
5. Write a formula and find the area of the following triangles.

Space for Your Work

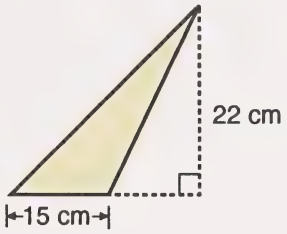
a.



b.



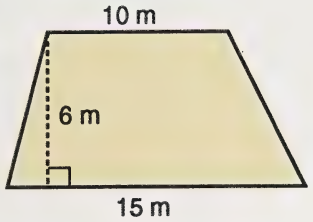
c.



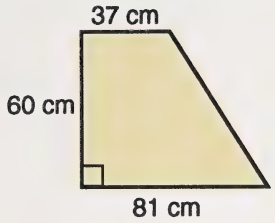
6. Write a formula and find the area of the following trapezoids.

Space for Your Work

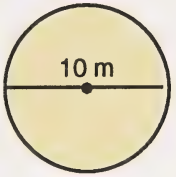
a.



b.



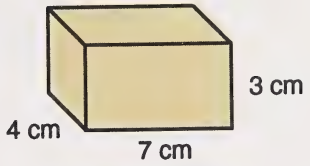
7. Write a formula and find the area of the circle.



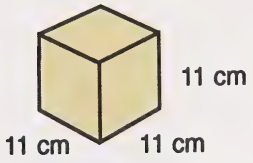
8. Write a formula and find the volume of the following right rectangular prisms.

Space for Your Work

a.



b.



See your learning facilitator to check your answers and to receive further instructions.

What lies Ahead

In this section you will learn this skill.

- using formulas as indirect measures of the perimeter of polygons

In this section you will use these words.

- rectangle
- parallelogram
- regular polygon

Working Together

Perimeter is the distance around a figure. Perimeter is a measurement of length.

Finding the perimeter of an object is a useful skill for everyday life. For example, to find how much wood you need to fence your property, you need to find the perimeter.

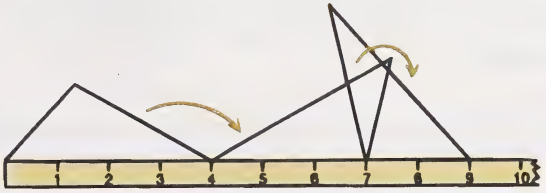
In the Introductory Activities you will find the perimeter of figures which you will cut out of the Appendix.

Introductory Activities

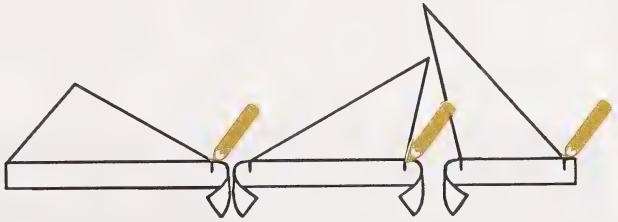
Space for Your Work

1. Cut out the figures labelled *Section 13 Polygons* in the Appendix. Then use one or more of the following methods to find the perimeter of each figure. Record the perimeter of each figure on the figure.

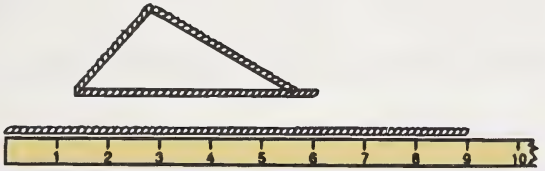
   a. Find the perimeter of the figures by "rolling" the figure along a metric ruler or metre stick.



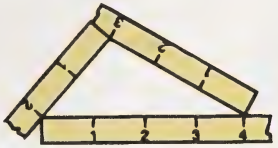
   b. Use a strip of paper to help you find the perimeter of each figure. Then measure the strips of paper and find the sum.



c. Use string to help you find the perimeter of each figure. Then measure the string.



d. Measure each side of a figure and find the sum.

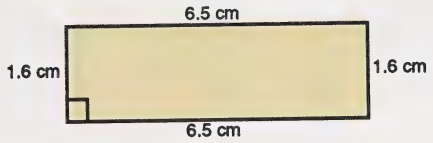


Space for Your Work

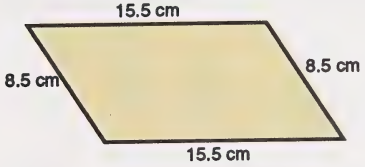
2. Find the perimeter of each of the following figures.

Space for Your Work

a.



b.



c.

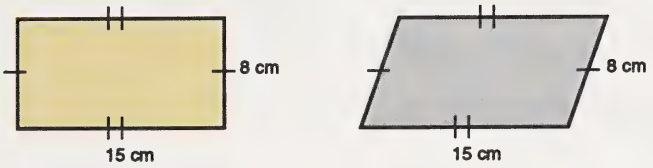
See your learning facilitator to check your answers and to receive further instructions.

Working Together

Perimeter of Rectangles and Parallelograms

The opposite sides of rectangles and parallelograms are equal. This property helps you to calculate the perimeter when you are given the lengths of only two sides.

Example



$$\begin{aligned} P &= 15 + 15 + 8 + 8 \\ &= 2 \times 15 + 2 \times 8 \\ &= 30 + 16 \\ &= 48 \end{aligned}$$

The perimeter of both the rectangle and the parallelogram is 48 cm.

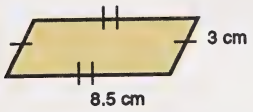
You can find the perimeter of any rectangle or parallelogram by doubling the length and the width and then adding the products.

This rule can be expressed by an equation or **formula**.

$$P = 2l + 2w$$

You can use the formula $P = 2l + 2w$ to find the perimeter of other rectangles and parallelograms without measuring all four sides.

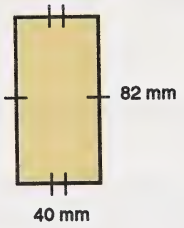
Example 1



$$\begin{aligned} P &= 2l + 2w \\ &= 2 \times 8.5 + 2 \times 3 \\ &= 17 + 6 \\ &= 23 \end{aligned}$$

The perimeter is 23 cm.

Example 2



$$\begin{aligned} P &= 2l + 2w \\ &= 2 \times 40 + 2 \times 82 \\ &= 80 + 164 \\ &= 244 \end{aligned}$$

The perimeter is 244 mm.

Perimeter of a Regular Polygon

A **regular polygon** is a polygon whose sides are line segments of equal length and whose angles are of equal measure. You can find the perimeter of a regular polygon when given the length of only one side.

Example

$$P = 7.2 + 7.2 + 7.2 + 7.2 + 7.2$$

$$= 5 \times 7.2$$

$$= 36$$

The perimeter of the regular pentagon is 36 cm.

You can find the perimeter of any regular polygon quickly by multiplying the number of equal sides by the length of one side.

The rule can be expressed by an equation or formula.

$$P = ns$$

n is the number of sides, s is the length of one side.

You can use the formula to find the perimeter of other regular figures.

Example 1

$$P = ns$$

$$= 4 \times 8.6$$

$$= 34.4$$

The perimeter of the square is 34.4 m.

Example 2

What is the perimeter of a regular decagon that has sides which measure 9.8 cm?

A regular decagon has ten equal sides.

$$P = ns$$

$$= 10 \times 9.8$$

$$= 98$$

The perimeter of the regular decagon is 98 cm.

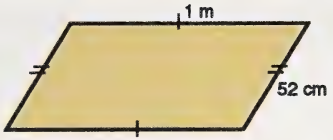
Note

Don't forget the units. Your answer is incomplete without them.

Converting Units

Sometimes the dimensions of a figure are measured in different units. To calculate the perimeter, you must first express the dimensions in the same unit.

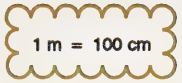
Example



This problem can be solved two ways.

Method 1

Express the length and width in centimetres.

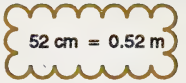


$$\begin{aligned} P &= 2l + 2w \\ &= 2 \times 100 + 2 \times 52 \\ &= 200 + 104 \\ &= 304 \end{aligned}$$

The perimeter is 304 cm.

Method 2

Express the length and width in metres.

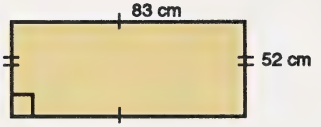


$$\begin{aligned} P &= 2l + 2w \\ &= 2 \times 1 + 2 \times 0.52 \\ &= 2 + 1.04 \\ &= 3.04 \end{aligned}$$

The perimeter is 3.04 m.

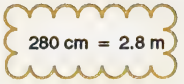
Sometimes you wish to change the units after you have calculated the perimeter.

Example



$$\begin{aligned} P &= 2l + 2w \\ &= 2 \times 83 + 2 \times 52 \\ &= 166 + 104 \\ &= 280 \end{aligned}$$

The perimeter is 280 cm.



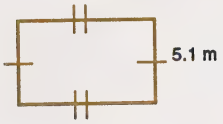
The perimeter is 2.8 m.

Practice Activities

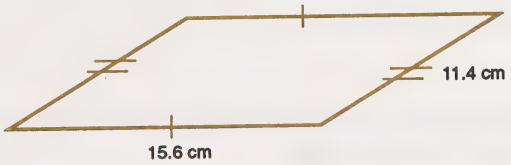
Space for Your Work

1. Write a formula and then find the perimeter of each polygon.

a.



b.



c.

d.

e.

2. Write a formula for the perimeter and find the perimeter of each of these polygons.

   a. a square with sides that are 9.2 m long

   b. a regular pentagon with sides that are 73 mm long

   c. a regular nonagon with sides that are 18 cm long

3. Use formulas to find the perimeters of the following polygons.

   a. a rectangle with dimensions of 0.4 m by 1600 mm

   b. a parallelogram with dimensions of 14 cm by 150 mm

   c. a rectangle with a length of 8.1 cm and a width of 45 mm

   d. a rectangle with a base of 14 mm and a height of 23 mm

   e. a rectangle with dimensions of 71 mm and 18 mm

Space for Your Work

4. The Pentagon in Washington, D.C. is so named because of its shape. Each of its outer walls is 302 m long. Find the minimum distance (in kilometres) that you would travel while walking around the outside of the Pentagon.

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

Extra Practice

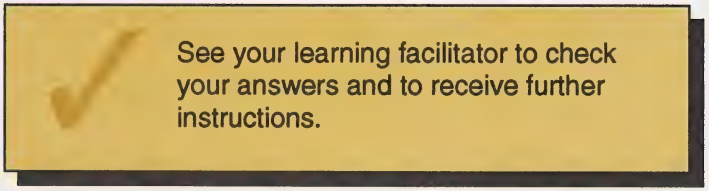
Space for Your Work

Complete the puzzle on the following page.[1]

First, figure out the perimeter of any polygon. Then find your answer in the coded line at the bottom of the page.

Second, each time the answer appears in the code, write the letter of that problem above it.

Keep working until you have decoded the line.



See your learning facilitator to check your answers and to receive further instructions.

[1] Creative Publications for excerpt from *Mathimagination* ©1973, Sunnyvale, California 94086

CODE LINE

U	Triangle with sides of 13 cm, 17 cm, and 24 cm
M	Square with side of 20 cm
T	Rectangle with sides of 29 cm and 36 cm
G	Equilateral triangle with side of 43 cm
O	Parallelogram with sides of 8 cm and 18 cm
E	Regular octagon with side of 14 cm
I	Quadrilateral with sides of 23 cm, 29 cm, 31 cm, and 44 cm
D	Regular pentagon with side of 15 cm
S	Isosceles triangle with base of 12 cm and side of 19 cm
H	Rhombus with side of 55 cm
P	Rectangle with sides of 7 cm and 16 cm
A	Regular hexagon with side of 6 cm
N	Pentagon with sides of 13 cm, 14 cm, 17 cm, 22 cm, and 27 cm
R	Parallelogram with sides of 48 cm and 66 cm

Title: Wet Threat

228 cm 36 cm 127 cm 93 cm 50 cm 54 cm 228 cm 112 cm 46 cm 54 cm 130 cm 50 cm 36 cm

75 cm 36 cm 80 cm 46 cm 112 cm 228 cm 52 cm 93 cm 130 cm 220 cm 127 cm 93 cm 129 cm 50 cm

Concluding Activities

Space for Your Work

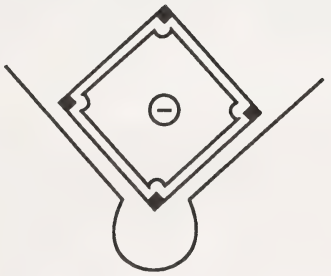
1. A window measures 124 cm by 92 cm.

   a. Find the length of the weather stripping needed to go around the window.

   b. The weather stripping is sold by the metre only. It costs 38¢/m. Find the cost of the weather stripping.

2. Each side of a stop sign is 28.2 cm long. Find the perimeter in metres.

3. The distance between each base on a ball diamond is 27.4 m. Find the approximate distance that a player travels to score a run.



4. A rectangular pool is 25 m long and 15 m wide. It is surrounded by a cement deck that is 1.5 m wide.

   a. Find the perimeter of the pool.

   b. Find the outside perimeter of the deck.

5. If fence posts are placed 2 m apart, how many are needed to fence a yard that is square with sides 16 m long?

6. Obtain a one dollar coin. The shape is classified as a hendecagon. It has eleven sides. First estimate its perimeter. Then measure its actual perimeter.

Space for Your Work

7. Mr. Wilson is building the frames for seven windows for a new house that he is building. The dimensions of the windows are as follows:

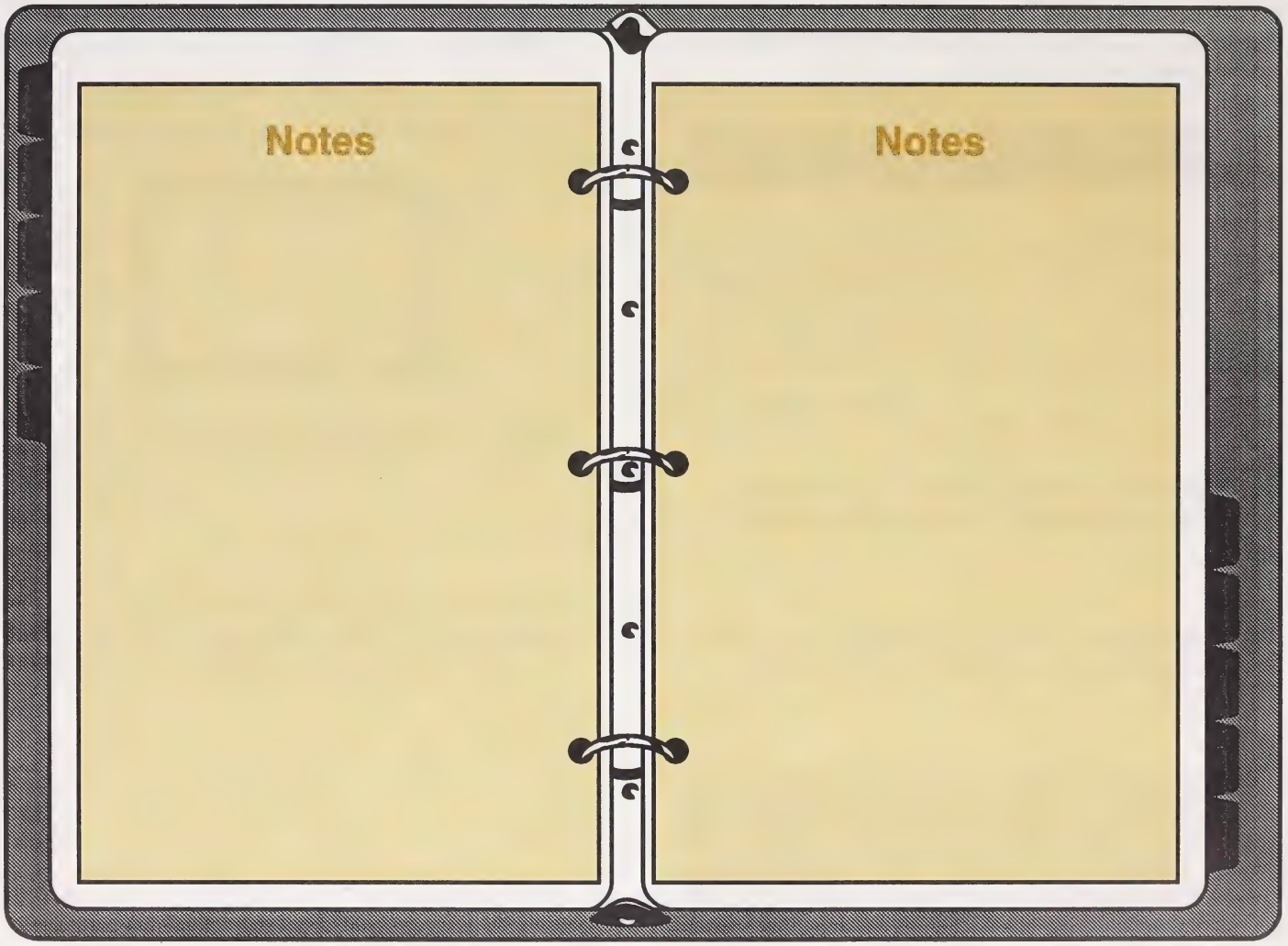
Number of Windows	Dimensions
2	1.5 m by 2 m
3	1.2 m by 1.5 m
1	2.5 m by 2 m
1	1.8 m by 2 m

a. How many metres of lumber does he need? (You may disregard the width of the lumber.)

b. If the lumber comes in 3 m lengths, what is the minimum numer of pieces that Mr. Wilson needs?

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.


Notes
Notes

What Lies Ahead

In this section you will learn these skills.

- knowing what π is and determining its value
- writing and using a formula that will indirectly measure the circumference of a circle

In this section you will use these words.

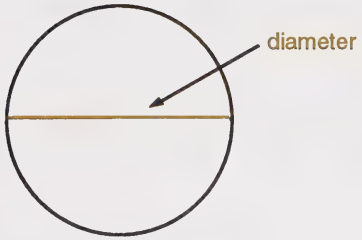
- pi
- diameter
- radius
- circumference

Working Together

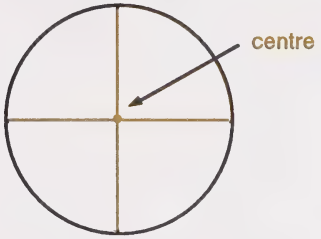
In the previous section you found the perimeters of polygons. In this section you will be finding the perimeters of circles.

The perimeter of a circle is called the **circumference**.

If you cut out a circle and fold it in half, the line segment formed is called the **diameter** of the circle.

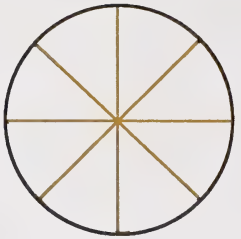


If you fold the circle in half again, another diameter is formed.

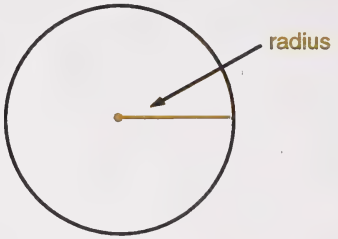


The point where the two diameters cross is called the **centre** of the circle.

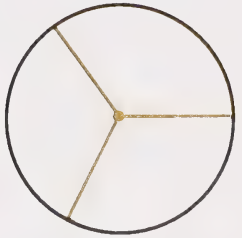
All the diameters of a circle are the same length.



A line segment from the centre of a circle to the outside of the circle is called the **radius** of the circle.



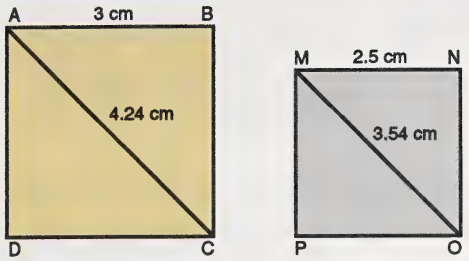
All the radii (plural of radius) of a circle are the same length.



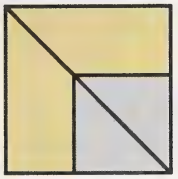
Introductory Activities

Space for Your Work

1. All squares are similar figures. They all have the same shape.



If you stack the figures, their diagonals will line up.



a. Calculate the ratio of the measures of $\overline{AB}$ to $\overline{MN}$.

b. Calculate the ratio of the measures of $\overline{AC}$ to $\overline{MO}$. Round to nearest tenth.

c. Calculate the ratio of the perimeter of square ABCD to square MNOP.

d. What do you notice about the ratios of the corresponding parts?

2. Use the squares in Question 1 to do the following questions.

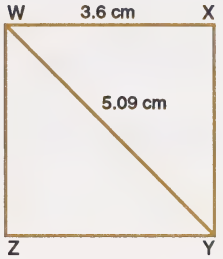
Space for Your Work

a. Calculate the ratio of the perimeter of square ABCD to the measure of its diagonal. Round to the nearest tenth.

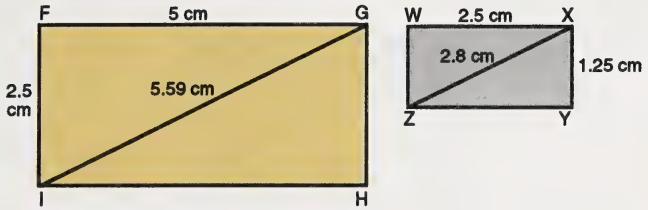
b. Calculate the ratio of the perimeter of square MNOP to the measure of its diagonal. Round to the nearest tenth.

c. What do you notice about the ratio of the perimeter to the measure of the diagonal of these squares?

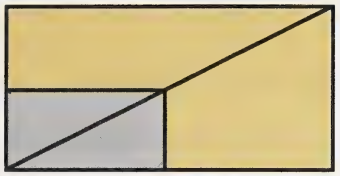
d. What do you think will be the ratio of the perimeter to the diagonal of this square?



3. Not all rectangles are similar. However, the following rectangles are similar figures.



If you stack the figures, their diagonals will line up.



a. Calculate the ratio of the perimeter of rectangle FGHI to the measure of its diagonal. Round to the nearest tenth.

b. Calculate the ratio of the perimeter of rectangle WXYZ to the measure of its diagonal. Round to the nearest tenth.

c. What do you notice about the ratios of these figures?

d. What do you think the ratio of perimeter to diagonal will be for other rectangles of this shape?

Space for Your Work

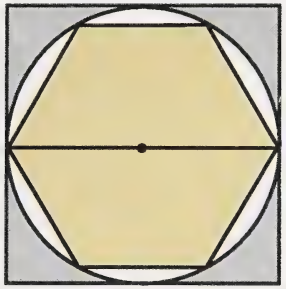
4. Measure the circumference and the diameter of four circular objects to the nearest tenth of a centimetre. Be as accurate as possible. Then calculate the ratio of the circumference to the diameter. Express the ratio as a decimal number. Display your data in the chart.

Space for Your Work

Object	Circum-ference (cm)	Diameter (cm)	Ratio of Circum-ference to Diameter

The ratio of the circumference of a circle to its diameter is about _____.

5. In the following diagram the diameter of the circle is 2 cm, the sides of the square are 2 cm, the sides of the hexagon are 1 cm.



Space for Your Work

a. Calculate the perimeter of the square.

b. Calculate the perimeter of the hexagon.

c. Estimate the perimeter of the circle.

See your learning facilitator to check your answers and to receive further instructions.

Working Together

As you have seen, there is a relationship between the circumference and the diameter of circular objects.

The ratio, $\frac{C}{d}$, is the same for all circles, regardless of their size. (Your quotients will have varied somewhat because of the inaccuracy of the devices used in measuring.) This ratio is represented by the Greek letter π (pi).

$$\frac{C}{d} = \pi$$

π is a constant value that has an infinite number of decimal places.

$$\pi = 3.141\,59\ldots$$

π is usually rounded off to the nearest hundredth.

$$\pi \doteq 3.14$$

Since $\frac{C}{d} = \pi$,

$$C = \pi d$$

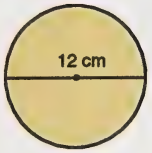
Since $d = 2r$,

$$C = \pi \times 2r$$

$$= 2\pi r$$

You can use these formulas to indirectly find the areas of circles.

Example 1



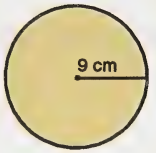
$$C = \pi d$$

$$= 3.14 \times 12$$

$$= 37.68$$

The circumference is 37.7 cm (rounded to the nearest tenth).

Example 2



$$C = 2\pi r$$

$$= 2 \times 3.14 \times 9$$

$$= 56.52$$

The circumference is 56.5 cm (rounded to the nearest tenth).

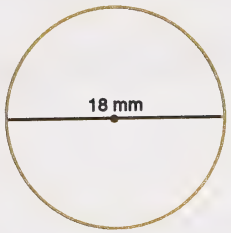
Practice Activities

Space for Your Work

1. **Estimate** the circumference of each circle with the following measurements.
   a. diameter is 5 cm
   b. radius is 9.5 m
   c. diameter is 20 cm
   d. radius is 15 mm

2. Calculate the circumferences of these circles. Round your answers to the nearest tenth.

   a.

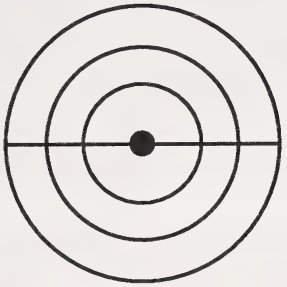


   b.

3. What is the circumference of a circular flower garden with a diameter of 4.2 m?

4. A golf ball has a diameter of 42.7 mm. What is its circumference?

5. The diameters of the three larger rings on a sheet of curling ice are 3.66 m, 2.44 m, and 1.22 m. Calculate the circumference of each ring.



Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

Extra Practice

Space for Your Work

Complete the puzzle on the following page.[1]

See your learning facilitator to check your answers and to receive further instructions.

[1] Creative Publications for excerpt from *Mathimagination* ©1973, Sunnyvale, California 94086

FIND A MATCH

Each of the two blocks below is divided into 20 boxes. Boxes in the top block contain the diameter (*d*) or radius (*r*) of a circle. Figure out the circumference (*C*) of any of these circles, using $\pi = 3.14$. Then find your answer in the bottom block. Transfer the word from the top box into the bottom box.

Keep working and you will spell out a funny saying.

$d = 1$ m STILL	$d = 1$ cm OR	$d = 3$ m TO	$d = 8$ m LOT	$d = 10$ cm A
$d = 5$ cm THE	$d = 12$ m OF	$d = 1.4$ cm ARE	$d = 9$ cm A	$d = 4.9$ cm NOT
$r = 1$ m WAS	$r = 3$ m FAMOUS	$r = 8$ cm WHETHER	$r = 10$ cm DECIDE	$r = 5$ m SCIENTISTS
$r = 50$ cm CRACK	$r = 1.5$ cm TRYING	$r = 3.3$ m ATOM	$r = 2.5$ m WISE	$r = 5.3$ m SPLITTING

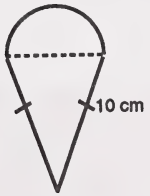
$C = 28.26$ cm	$C = 25.12$ m	$C = 37.68$ m	$C = 18.84$ m	$C = 31.4$ m
$C = 4.396$ cm	$C = 3.14$ m	$C = 9.42$ cm	$C = 9.42$ m	$C = 62.8$ cm
$C = 50.24$ cm	$C = 3.14$ cm	$C = 15.386$ cm	$C = 33.284$ m	$C = 15.7$ cm
$C = 20.724$ m	$C = 6.28$ m	$C = 31.4$ cm	$C = 15.7$ m	$C = 314$ cm

Concluding Activities

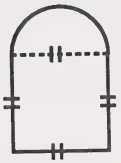
Space for Your Work

1. Find the perimeter of each figure. Round to the nearest tenth.

a.



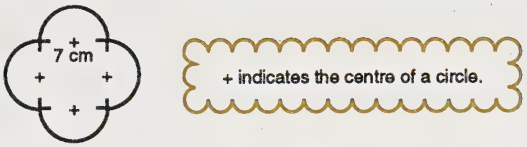
b.



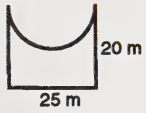
c.

Space for Your Work

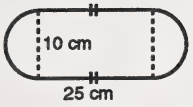
d.



e.



f.



2. The diameter of the earth at the equator is approximately 12 750 km.

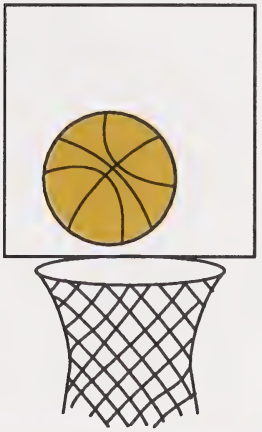
   a. The circumference of the earth is about how many kilometres?

   b. If a satellite is orbiting the earth 36 000 km above the earth's surface, how far does it travel in completing one orbit?

3. The diameter of a basketball is 24.5 cm. The diameter of a basketball hoop is 45 cm. (You may wish to draw a diagram to help you answer the questions.)

   a. The circumference of the hoop is how much larger than the circumference of the basketball?

   b. If the ball goes through the center of the hoop, find the distance between the ball and the hoop.



Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

What Lies Ahead

In this section you will learn this skill.

- using a formula to determine the area of a rectangle

In this section you will use these words.

- area
- square units
- base
- height

Working Together

Area is used in many everyday situations, such as buying carpet for your livingroom or sod for your lawn.

Area is a measurement of the surface that a figure contains. Square units are used to measure area.

square unit

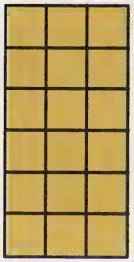
Example

The area of the rectangle is 6 square units.

Introductory Activities

Space for Your Work

1. Find the area of each rectangle.

   a. 

   b. 

   c. 

2. a. Did you count squares to find the area?

   b. Is there an easier way?

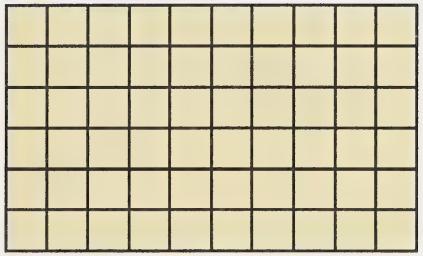
See your learning facilitator to check your answers and to receive further instructions.

Working Together

Formula for the Area of a Rectangle

As you discovered, you do not need to count all the square units to find the area of a rectangle. You can simply count the number of square units in a row and multiply by the number of rows.

Example



There are ten square units in one row. There are six rows.

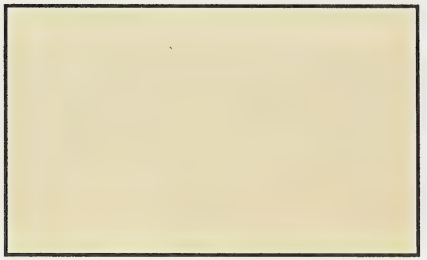
$$10 \times 6 = 60$$

So, there are sixty square units in the rectangle. In other words, the area of the rectangle is 60 square units.

Since the square unit in the example is 1 cm^2, the area of the rectangle is 60 cm^2. You can describe this relationship using words.

The number of square units equals the number of square units in a row times the number of rows.

If the rectangle was not divided into square units, you could still find the area if you knew the dimensions of the rectangle.



The number of square units in one row corresponds to the length of the base of the rectangle. The number of rows corresponds to the height of the rectangle.

This can be expressed with a formula.

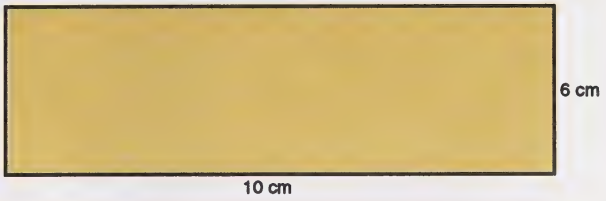
Area of a rectangle = base $\times$ height

or

$A = bh$

You can use this formula to find the area of rectangles without having to measure all four sides.

Example



$A = b \times h$

$= 10 \times 6$

$= 60$

The area is 60 cm^2.

Note

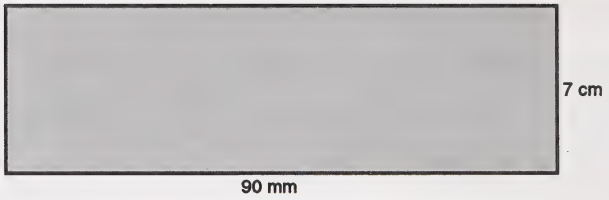
Be careful to include the correct units.

Area is measured in square units.

60 cm^2 is read as "60 square centimetres."

Both the base and the height must be measured in the same unit before you calculate the area.

Example



90 mm = 9 cm

$A = b \times h$

$= 9 \times 7$

$= 63$

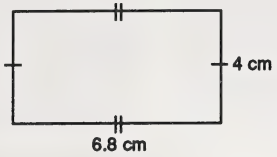
The area is 63 cm^2.

Practice Activities

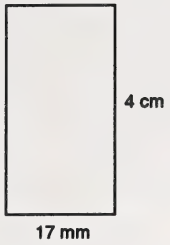
Space for Your Work

1. Use the formula to find the area of each rectangle.

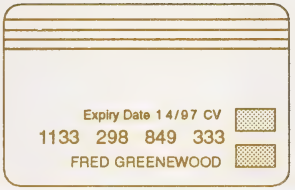
   a.



   b.



2. A credit card measures 8.5 cm by 5.4 cm. Find its area.



See your learning facilitator to check your answers and to receive further instructions.

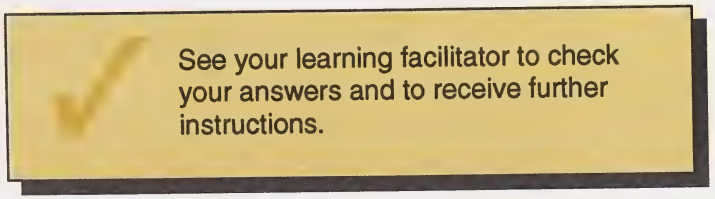
Extra Practice

Space for Your Work

Complete the puzzle on the following page.[1]

Figure out the area of each rectangle. Then write the letter inside each rectangle into a box at the bottom of the page. The letter of the smallest rectangle goes in the first box, the letter of the next smallest rectangle goes in the second box, and so on up to the largest rectangle.

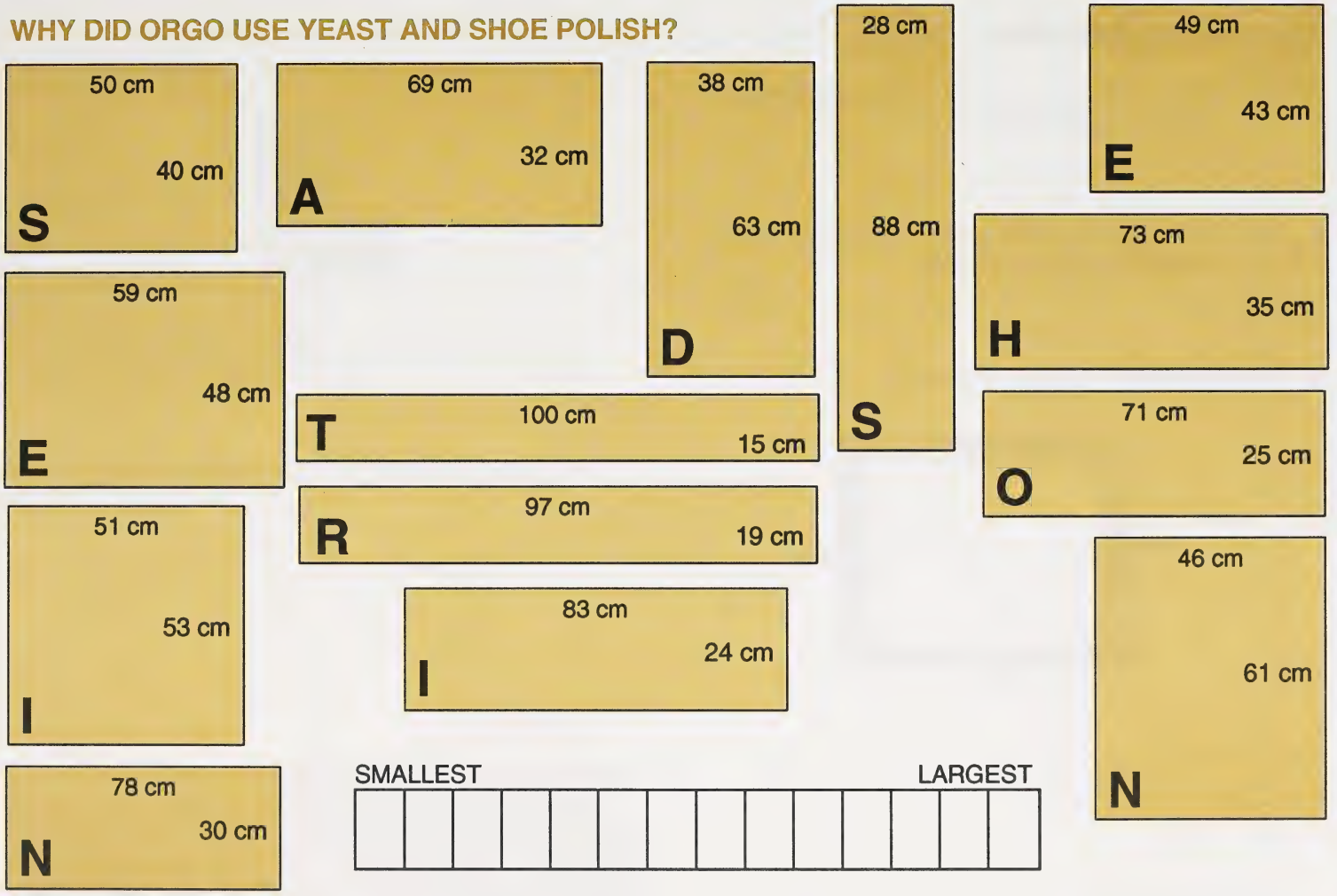
Write the letters in proper order and you will have the answer to the question.



See your learning facilitator to check your answers and to receive further instructions.

[1] Creative Publications for excerpt from *Mathimagination* ©1973, Sunnyvale, California 94086

WHY DID ORGO USE YEAST AND SHOE POLISH?



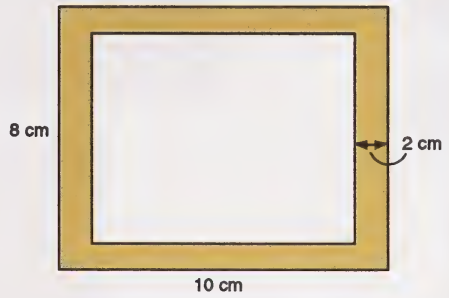
Concluding Activities

Space for Your Work

1. a. What happens to the area of a rectangle when its sides are doubled in length?

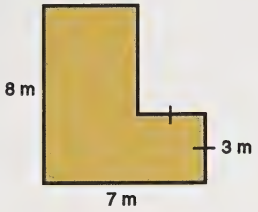
   b. What happens to the area of a rectangle when its sides are tripled in length?

2. Find the area of this picture frame.



3. Mrs. Ben Zui is carpeting her living room and dining room. The cost of carpeting is $\$34.24/m^2$.

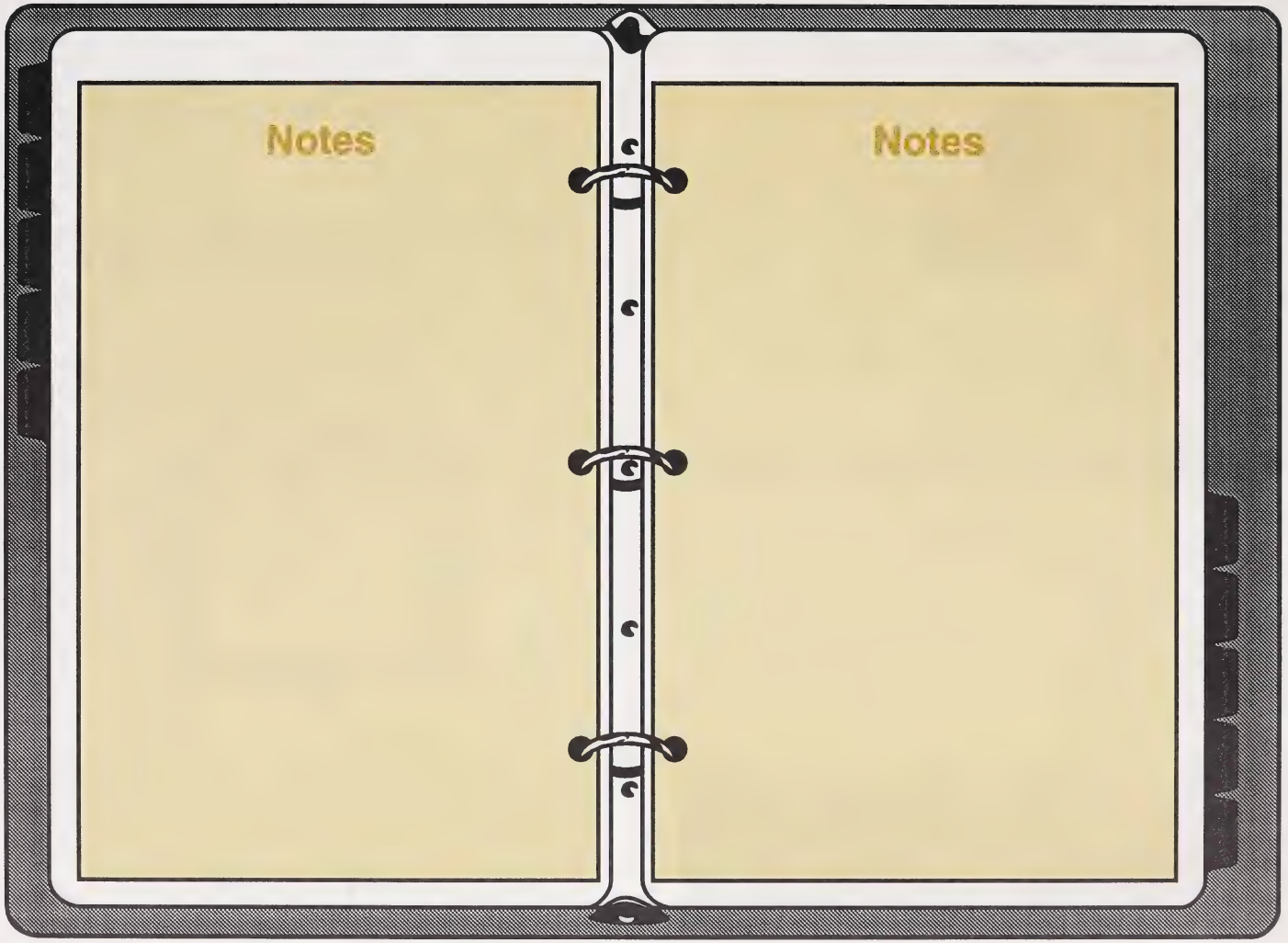
Space for Your Work



a. Find the amount of carpet needed.

b. Calculate the cost of carpeting the two rooms.

See your learning facilitator to check your answers and to receive further instructions.


Notes
Notes

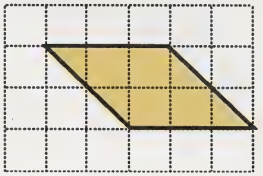
What Lies Ahead

In this section you will use this skill.

- using a formula to determine the area of a parallelogram

Working Together

It is difficult to accurately find the area of a parallelogram by counting square units.

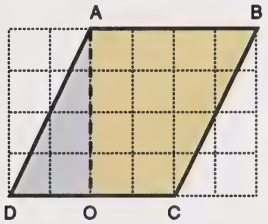


In this section you will learn an indirect method of finding the area of a parallelogram.

Introductory Activities

Space for Your Work

1. Draw figure ABCD on a sheet of grid paper from the Appendix. Cut off the triangular portion AOD and slide it across the figure so $\overline{AD}$ fits on $\overline{BC}$. Then find the area of figure ABCD.

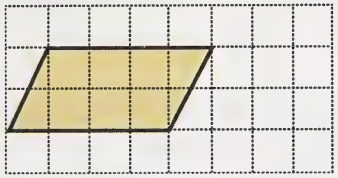


2. Use the method that you discovered in Question 1 to find the areas of the following figures.

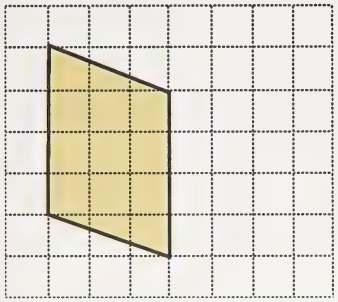
Space for Your Work

a.

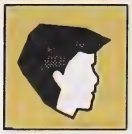
b.



c.



See your learning facilitator to check your answers and to receive further instructions.

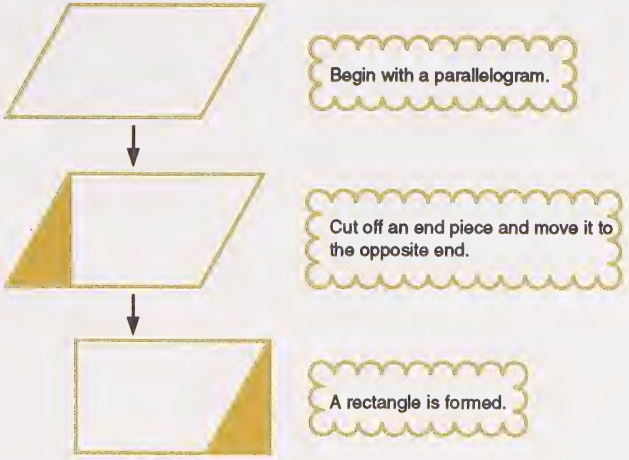


Working Together

Area of a Parallelogram

In the Introductory Activities you discovered that a parallelogram is related to a rectangle.

To find the area of a parallelogram, you can cut off one of the triangular end pieces and slide it to the opposite end of the parallelogram. The new figure that is formed is a rectangle.



Notice that neither the length of the base nor the height changes. Therefore, the formula for the area of a rectangle can be used to find the area of a parallelogram.

Area of parallelogram = base × height

or

$A = b \times h$

Note

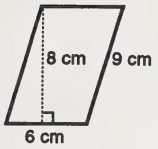
Always measure the height of a parallelogram at a right angle, or **perpendicular**, to the base.

Practice Activities

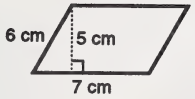
Space for Your Work

1. What is the length of the base and the height of each parallelogram?

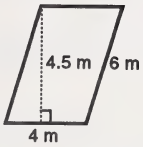
   a.



   b.



   c.



2. Use the formula to find the area of each parallelogram in Question 1.

3. Complete the chart for parallelograms.

Space for Your Work

	Base	Height	Area in Smaller Units	Area in Larger Units
a.	25 cm	80 mm		
b.	800 cm	5 m		
c.	4.2 cm	32 mm		
d.	0.2 km	800 m		

See your learning facilitator to check your answers and to receive further instructions.

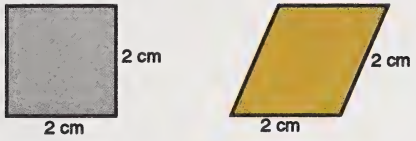
Concluding Activities

Space for Your Work

1. The base of a parallelogram is 10 cm. The height is 2 cm more than half the base. Find the area of the parallelogram.

2. The height of a parallelogram is 4.5 cm. The base is twice the height. What is the area of the parallelogram?

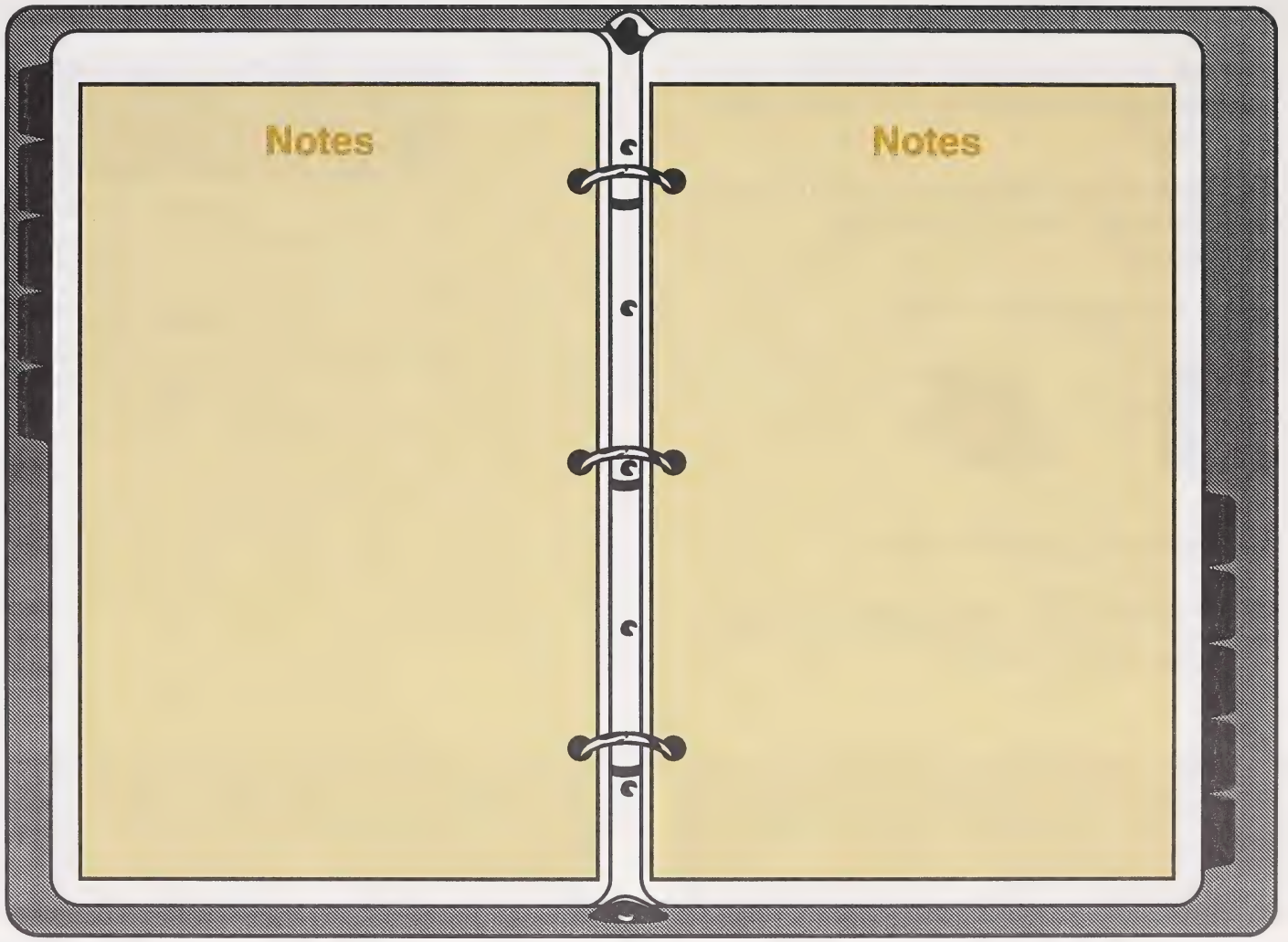
3. Below are a square and a rhombus.



   a. What is the area of the square?

   b. Is the area of the rhombus greater than, less than, or equal to the area of the square? Give a reason for your answer.

See your learning facilitator to check your answers and to receive further instructions.


Notes
Notes

What Lies Ahead

In this section you will learn these skills.

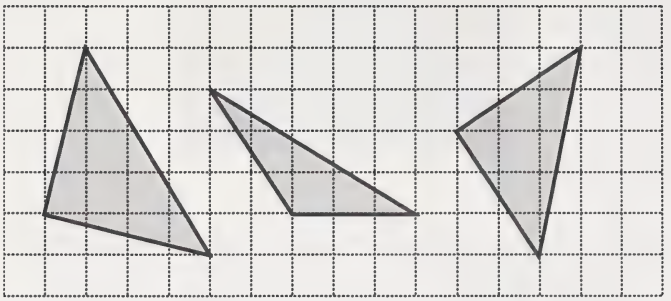
- using a formula to determine the area of triangles
- multiplying and dividing with linear and area measurements

In this section you will use these words.

- base of a triangle
- height of a triangle

Working Together

It is difficult to accurately find the area of a triangle by counting squares.

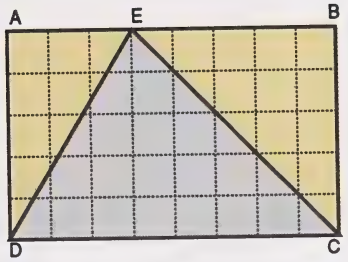


In this section you will learn an indirect method of finding the area of triangles.

Introductory Activities

Space for Your Work

1. Draw rectangle ABCD on a sheet of grid paper from the Appendix. Cut out the rectangle and then cut out triangle CDE as illustrated.



Now arrange the two smaller triangles, triangle AED and triangle EBC, to fit exactly inside triangle CDE. You may have to turn or flip the pieces to make them fit.

a. How is the area of triangle CDE related to the area of rectangle ABCD?

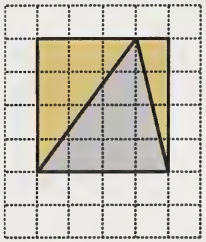
b. Calculate the area of the rectangle.

c. Use the relationship you discovered in part a. to find the area of triangle CDE.

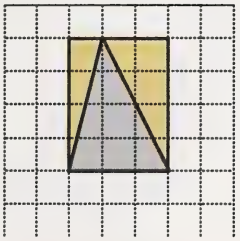
2. Use the method you found in Question 1 to find the area of triangle ABC in each of the following.

Space for Your Work

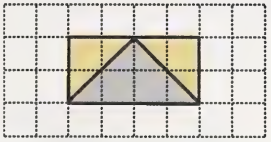
a.



b.



c.



See your learning facilitator to check your answers and to receive further instructions.

Working Together

Area of Triangles

In the previous activity you found that the area of each triangle was half of the area of the rectangle of which the triangle was a part.

Therefore, the area of the triangle can be found by finding one half of the area of the rectangle.

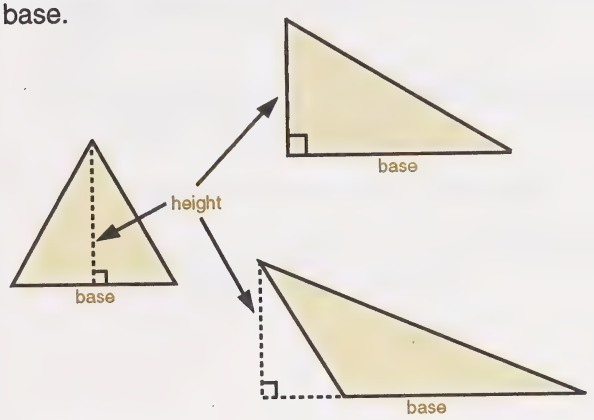
$$\text{Area of triangle} = \frac{\text{base} \times \text{height}}{2}$$

or

$$A = \frac{bh}{2}$$

Note

Remember that height is measured at a right angle to the base.



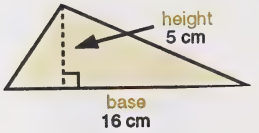
Use this formula to find the area of other triangles.

Example 1

$$A = \frac{b \times h}{2}$$

$$= \frac{16 \times 5}{2}$$

$$= 40$$



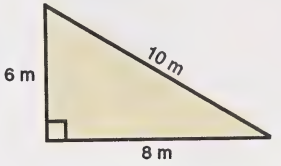
The area of the triangle is 40 cm^2.

Example 2

$$A = \frac{b \times h}{2}$$

$$= \frac{6 \times 8}{2}$$

$$= 24$$



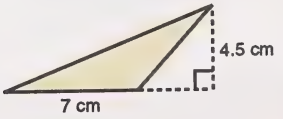
The area of the triangle is 24 m^2.

Example 3

$$A = \frac{b \times h}{2}$$

$$= \frac{7 \times 4.5}{2}$$

$$= 15.75$$



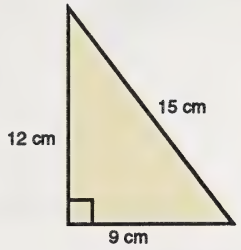
The area of the triangle is 15.75 cm^2.

Practice Activities

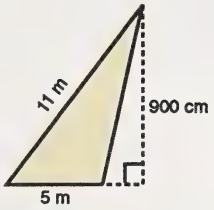
Space for Your Work

1. Use a formula to calculate the area of each triangle.

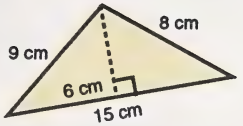
a.



b.



c.



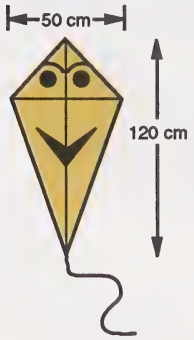
2. Complete the chart for triangles.

Space for Your Work

	Base	Height	Area
a.	15 m	6 m	
b.	3.4 cm	5 cm	
c.	9 m	900 cm	
d.	40 mm	5 cm	
e.	4 cm	6 cm	

Note the different units.

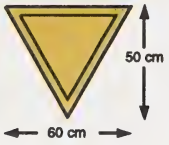
3. Calculate the area of the kite shown.



4. How much felt is needed to make 30 pennants?

Space for Your Work

5. Calculate the area of the traffic sign.



Computer Alternative

6. Do Lesson Series E in *Geometric Concepts/Area*.

See your learning facilitator to check your answers and to receive further instructions.

Extra Practice

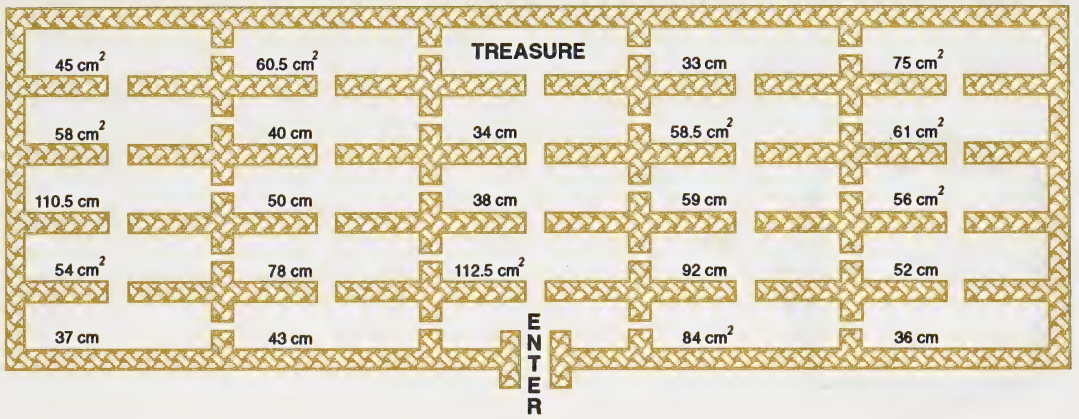
Space for Your Work

Do the puzzle on the following page.[1]

See your learning facilitator to check your answers and to receive further instructions.

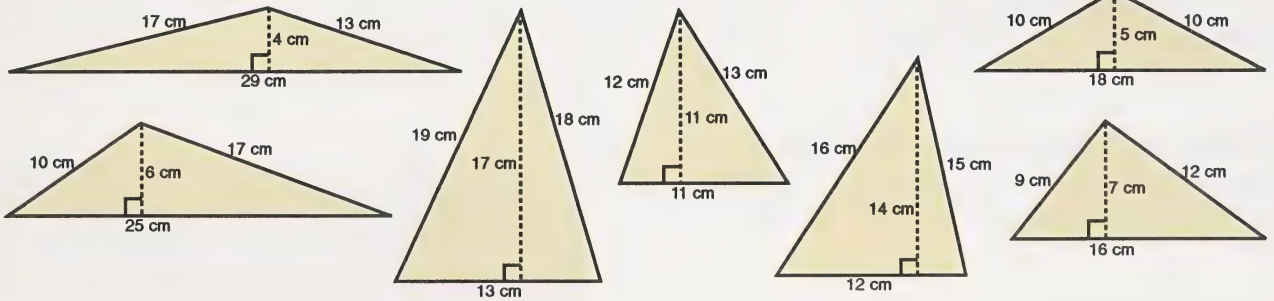
[1] Creative Publications for excerpt from *Mathimagination* ©1973, Sunnyvale, California 94086

Maze Daze



Each room in this maze contains a measurement. Fourteen of these measurements are the areas and perimeters of the seven triangles below. Figure out the area and perimeter of any triangle and find the answers in the maze. Circle the answers.

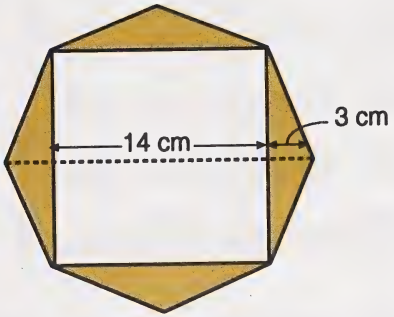
Keep working until you can draw a path to the treasure room that goes only through rooms containing correct answers. (It might not go through all of the correct answers.)



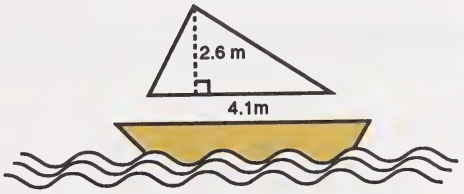
Concluding Activities

Space for Your Work

1. Find the area of the shaded part of the regular octagon.



2. Calculate the area of the sail.



3. Mr. Rawlins has a flower bed shaped like an isosceles right triangle in the corner of his lot. Find the area if the sides measure 2.4 m, 2.4 m, and 3.4 m.

4. What happens to the area of a triangle under the following conditions?

   a. the height is doubled

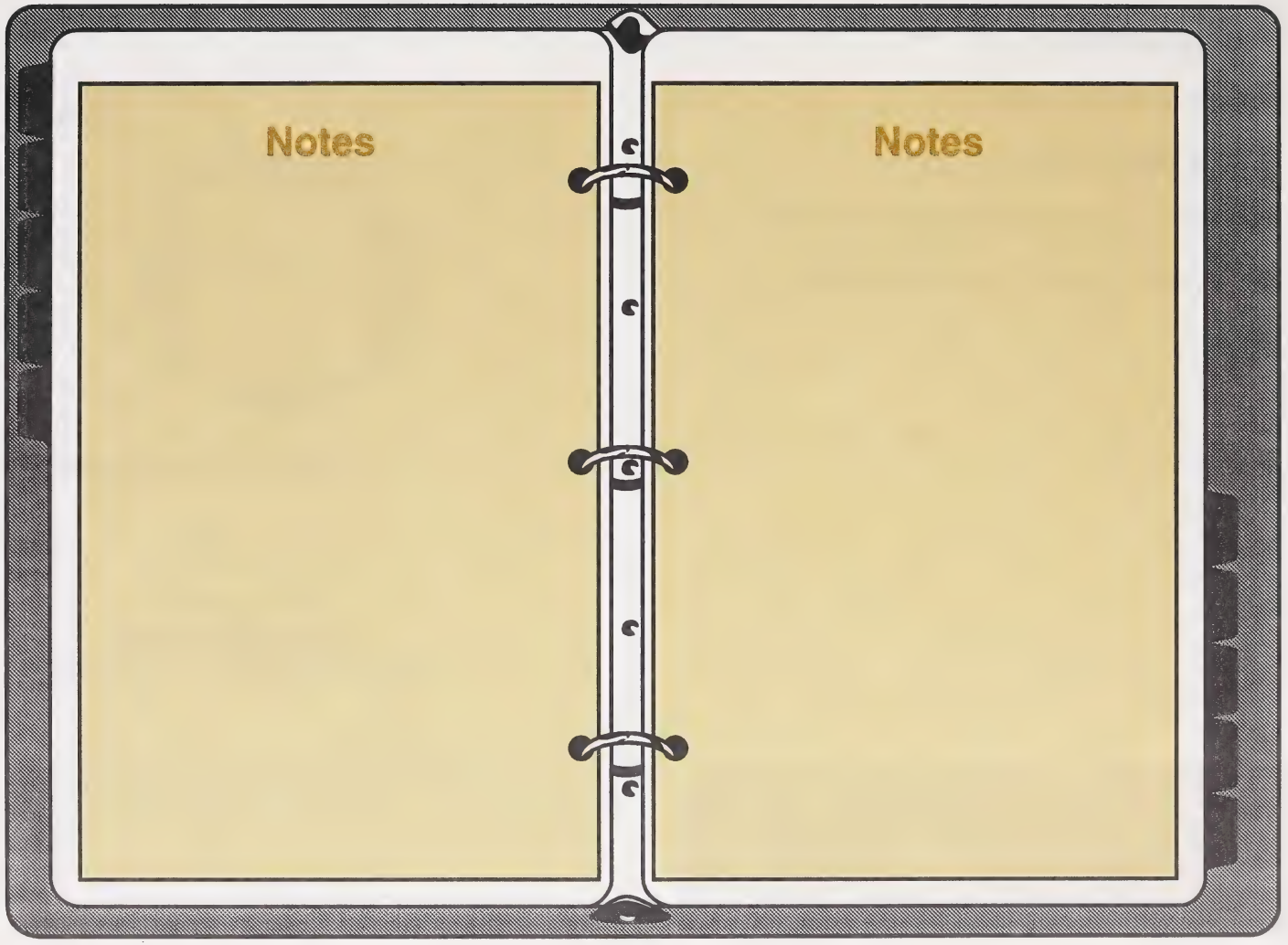
   b. the base is doubled

   c. both the base and the height are doubled

   d. both the base and the height are tripled

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.


Notes
Notes

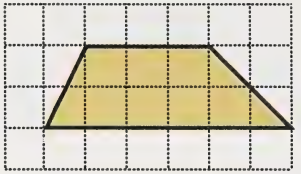
What Lies Ahead

In this section you will learn this skill.

- using a formula to determine the area of a trapezoid

Working Together

It is difficult to find the area of a trapezoid by counting the square units.

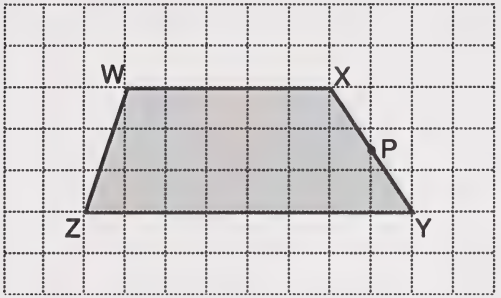


In this section you will learn an indirect method of finding the area of trapezoids.

Introductory Activities

Space for Your Work

1. Draw the following trapezoid on a piece of grid paper and then find the half turn image about point P.



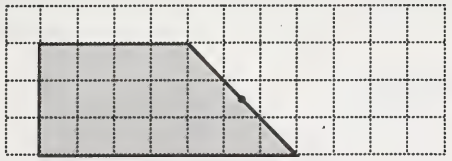
   a. What does the combination of the two figures (original trapezoid and image) look like?

   b. How do you find the area of this shape?

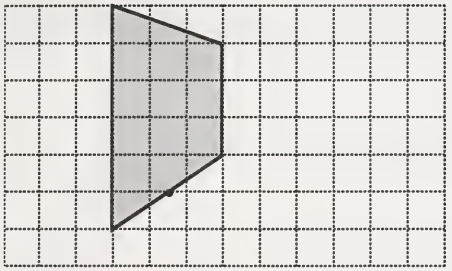
2. Use the method you discovered in Question 1 to find the area of these trapezoids.

Space for Your Work

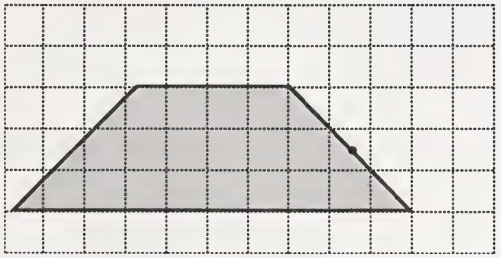
a.



b.



c.



See your learning facilitator to check your answers and to receive further instructions.

Working Together

Area of a Trapezoid

In the Introductory Activities you turned the trapezoids so that that figure and its image created a parallelogram.

You discovered that the area of the parallelogram was the product of its height (h) and its new base ($b_1 + b_2$).

$$\text{Area of parallelogram} = \text{height} \times \text{base}$$

or

$$\text{Area of parallelogram} = h\,(b_1 + b_2)$$

You also discovered that the area of the trapezoid was one half of the area of the parallelogram.

$$\text{Area of trapezoid} = \frac{\text{area of parallelogram}}{2}$$

or

$$A = \frac{h(b_1 + b_2)}{2}$$

You can use this formula to indirectly find the area of other trapezoids.

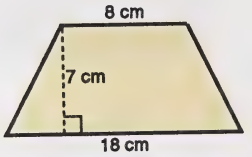
Example

$$A = \frac{h\,(b_1 + b_2)}{2}$$

$$= \frac{7\,(8 + 18)}{2}$$

$$= \frac{7 \times 26}{2}$$

$$= 91$$



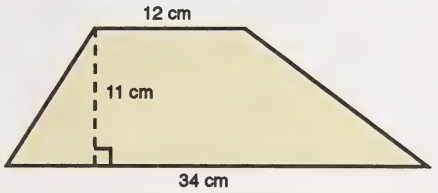
The area of the trapezoid is 91 cm^2.

Practice Activities

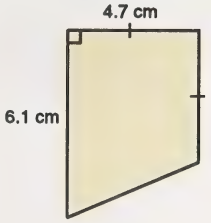
Space for Your Work

1. Use the formula to find the areas of these trapezoids.

   a.



   b.



2. Find the area of the sign.

3. Complete the chart for trapezoids.

Space for Your Work

	Height	$Base_1$	$Base_2$	Area
a.	6 cm	12 cm	10 cm	
b.	5.4 m	3.8 m	7.2 m	
c.	22 mm	3 cm	3.2 cm	
d.	5 m	4 m	6 m	

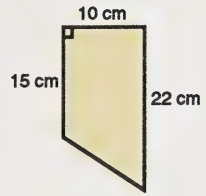
See your learning facilitator to check your answers and to receive further instructions.

Extra Practice

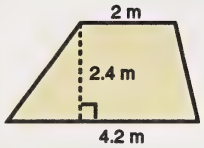
Space for Your Work

Use a formula to find the area of each trapezoid.

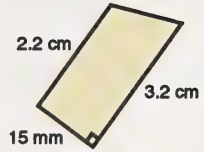
1.



2.



3.

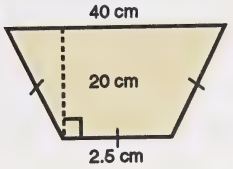


See your learning facilitator to check your answers and to receive further instructions.

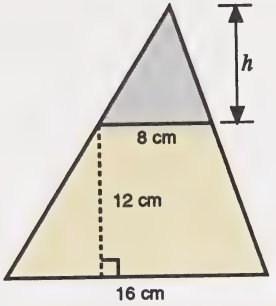
Concluding Activities

Space for Your Work

1. Mr. Williams has a flower garden that looks like a trapezoid. Find the area of the end of the flower garden.

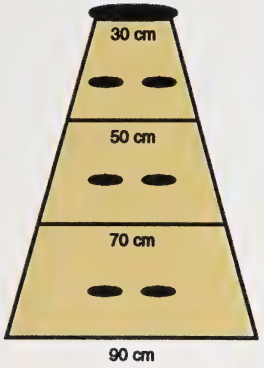


2. a. The top of a triangle was cut off. Find the area of the part that remains.



b. If the area of the original triangle was 180 cm^2, find the height (h) of the triangle that was cut off.

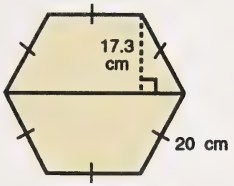
3. The side of a vaulting horse for gymnastics is shaped like a trapezoid. It is made up of three sections that lock together. Each section is 40 cm high.



   a. Find the area of each section.

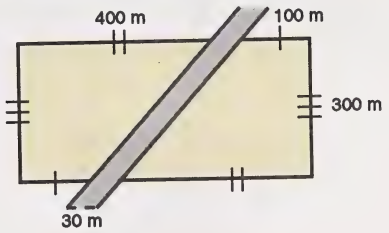
   b. What is the total area of the side of the vaulting horse?

4. Find the area of this hexagon.



Space for Your Work

5. A highway was built through Mr. Ingram's farm, dividing one of his fields as illustrated.



a. Find the area of one of the resulting fields (they're both the same).

b. What was the area of the original field?

c. How much land did he lose to the highway?

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

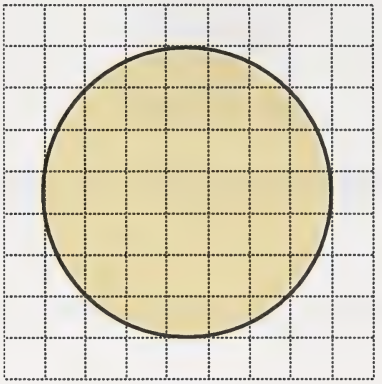
What Lies Ahead

In this section you will learn these skills.

- using a formula to determine the area of a circle

Working Together

It is difficult to find the area of a circle by counting square units.



In this section you will learn a method to indirectly find the area of a circle.

Introductory Activities

Space for Your Work

1. In the following diagram the diameter of the circle is 2 cm, the sides of the larger square are 2 cm, and the sides of the smaller square are 1.4 cm.

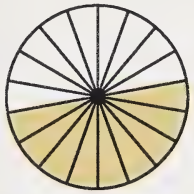
a. Calculate the area of the larger square.

b. Calculate the area of the smaller square.

c. Estimate the area of the circle.

2. Use your compass to draw a large circle. The radius should be at least 8 cm.

   Use your protractor to divide the circle into 18 equal parts of 20°. Colour the bottom half of the circle.



   Now cut out the parts and reassemble them like this.

   a. Calculate the area of the parallelogram formed.

   b. How are the parts of the parallelogram and circle related?

   c. What is the area of the circle?

See your learning facilitator to check your answers and to receive further instructions.

Space for Your Work

Working Together

As you discovered, there are some relationships that exist between the parallelogram and the circle.

By using the formula for the area of a parallelogram and making some substitutions, a formula for the area of a circle can be developed.

$$\text{Area of a parallelogram} = b \times h$$

The height of the parallelogram is equal to the radius of the circle, and the length of the base is half of the circumference of the circle.

$$\text{Area of circle} = \frac{\pi \times d}{2} \times r$$

$d = 2r$

$$\text{Area of circle} = \frac{\pi \times 2r}{2} \times r$$

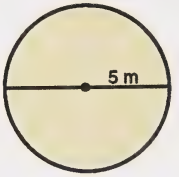
$$= \frac{\pi \times \overset{1}{\cancel{2}}r}{\underset{1}{\cancel{2}}} \times r$$

$$= \pi \times r \times r$$

$$= \pi \times r^2$$

You can use this formula to find the area of a circle indirectly.

Example



$$A = \pi \times r^2$$

$$A = 3.14 \times 5 \times 5$$

$$A = 78.5$$

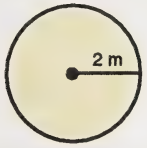
The area of the circle is 78.5 m^2.

Practice Activities

Space for Your Work

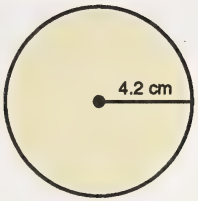
1. Use a formula to calculate the area of the following circles to the nearest tenth.

a.



b.

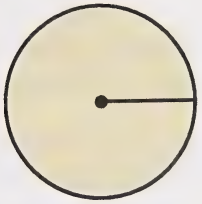
c.



2. Measure the radius of the circles in millimetres. Use the formula to calculate the areas to the nearest tenth.

Space for Your Work

a.



b.

3. A circular table has a diameter of 1.4 m. Find the area of the table top to the nearest tenth.

4. How much greater is the area of a quarter than the area of a dime?

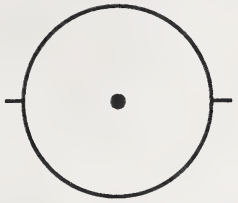
5. Estimate the areas of the circles which have the following radii.

   a. 10 cm

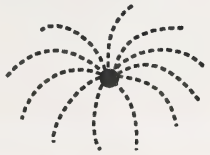
   b. 6 cm

   c. 20 m

6. A face-off circle has a radius of 4.5 m. Find its area.



7. A lawn sprinkler sprays water a distance of 3.2 m in all directions. Find the area of grass that will be watered.



Space for Your Work

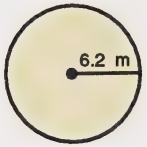
See your learning facilitator to check your answers and to receive further instructions.

Extra Practice

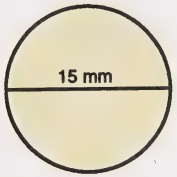
Space for Your Work

Use the formula to calculate the area of each circle to the nearest tenth.

1.



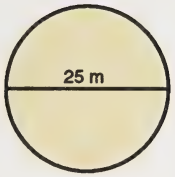
2.



3.

Space for Your Work

4.



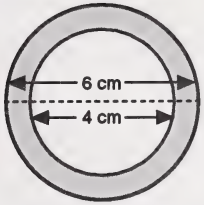
See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

Space for Your Work

1. Calculate the areas of the shaded regions.

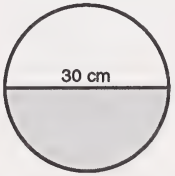
a.



b.

c.

d.



2. What is the best estimate of the area of a dime?

Space for Your Work

a. 10 mm^2

b. 30 mm^2

c. 300 mm^2

3. What is the best estimate of the area of a record?

a. 45 cm^2

b. 200 cm^2

c. 700 cm^2

4. What is the best estimate of the area of an archery target?

a. 1000 cm^2

b. 10 000 cm^2

c. 1 000 000 cm^2

5. Which gives you more pizza, two 20-cm pizzas or one 30-cm pizza?

6. The signal from a radio station transmitter can be received at locations 150 km away. Find the maximum area served by the radio station.

7. a. What happens to the area of a circle when its radius is doubled?

   b. What happens to the area of a circle when its radius is tripled?

Space for Your Work

8. A circular coffee table is made from oak veneer glued to a 2-cm thick plywood base. The veneer costs \$48.75/m^2 and the plywood costs \$10.95/m^2. The table has a diameter of 0.8 m.

   a. What is the area of the table top? (Round to the nearest tenth.)

   b. What is the cost of the veneer used to make the table top?

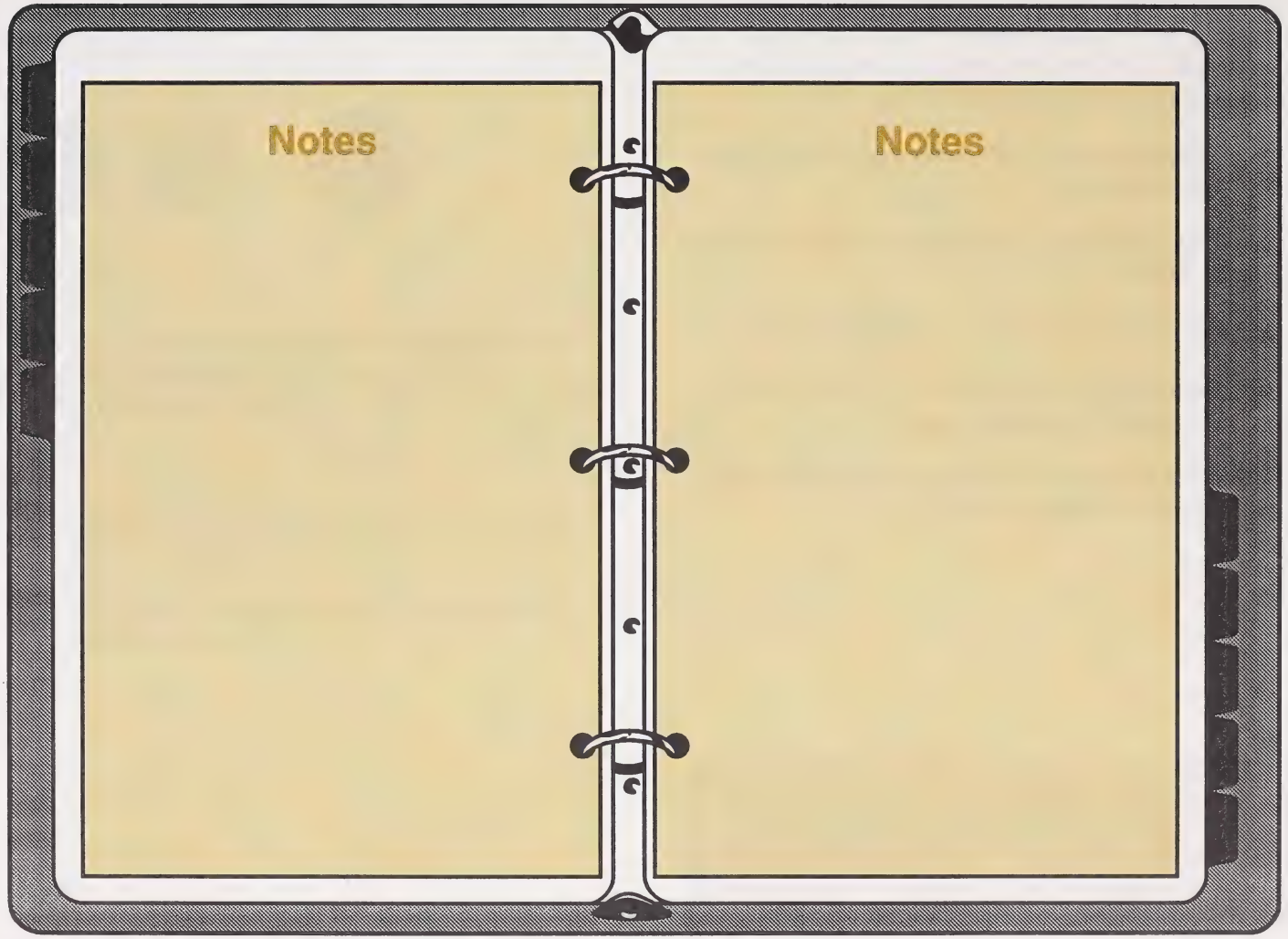
   c. What is the cost of the plywood used?

   d. If the table top is cut from a 1 m^2 piece, what is the value of the wasted veneer.

   e. What is the value of the wasted plywood under the same circumstances?

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.


Notes
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What Lies Ahead

In this section you will learn this skill.

- using a formula to find the volume of right rectangular prisms and cubes

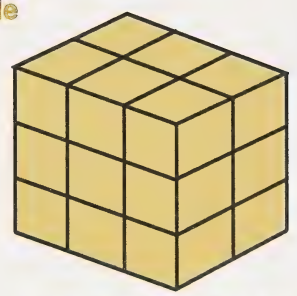
Working Together

Volume is a measurement of how much space an object occupies.

You can find volume by counting the number of cubic units.

1 cubic unit

Example

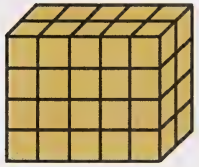


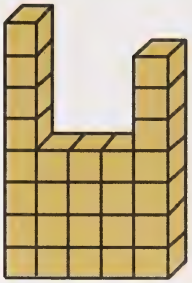
This right rectangular prism has 18 cubic units.

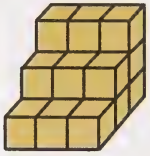
Introductory Activities

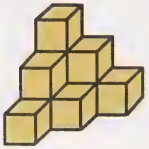
Space for Your Work

1. Find the volume of the following solids. You may use base 10 blocks or sugar cubes to construct the solids first.

   a. 

   b. 

   c. 

   d. 

2. Use twelve base 10 blocks or sugar cubes for this question.

   a. Stack the cubes to form a rectangular prism with one layer.

   b. Form a rectangular prism with two layers.

   c. Form a rectangular prism with three layers.

   d. Form a rectangular prism with four layers.

   e. Form a rectangular prism with six layers.

3. What is the volume of each of the rectangular prisms in Question 2?

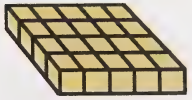
Space for Your Work

4. Use base 10 blocks or sugar cubes for this question.

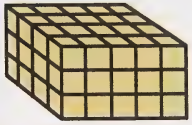
Space for Your Work

a. What is the volume of this right rectangular prism?

b. How can you find the volume of this right rectangular prism without counting each block? What is it?



c. How can you find the volume of this right rectangular prism without counting each block? What is it?



See your learning facilitator to check your answers and to receive further instructions.

Working Together

As you discovered, you can find the volume of a right rectangular prism by finding the number of blocks in the first layer (the base) and then multiplying your answer by the number of layers (the height).

You can express this rule with a formula.

$V = B \times h$

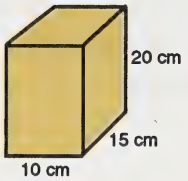
or

B = area of base

$V = l \times w \times h$

You can use this formula to find the volume of other rectangular prisms.

Example



$V = B \times h$

$= (10 \times 15) \times 20$

$= 3000$

The volume is 3000 cm^3.

Video Activity

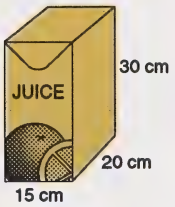
View the video *MATH WAYS: Volume* to discover how to find the volume of rectangular prisms.

Practice Activities

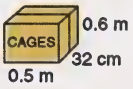
Space for Your Work

Calculate the volume of each of the following right rectangular prisms using a formula.

1.



2.



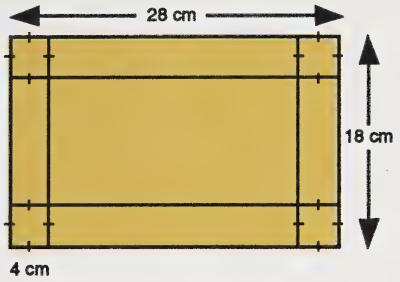
3.

See your learning facilitator to check your answers and to receive further instructions.

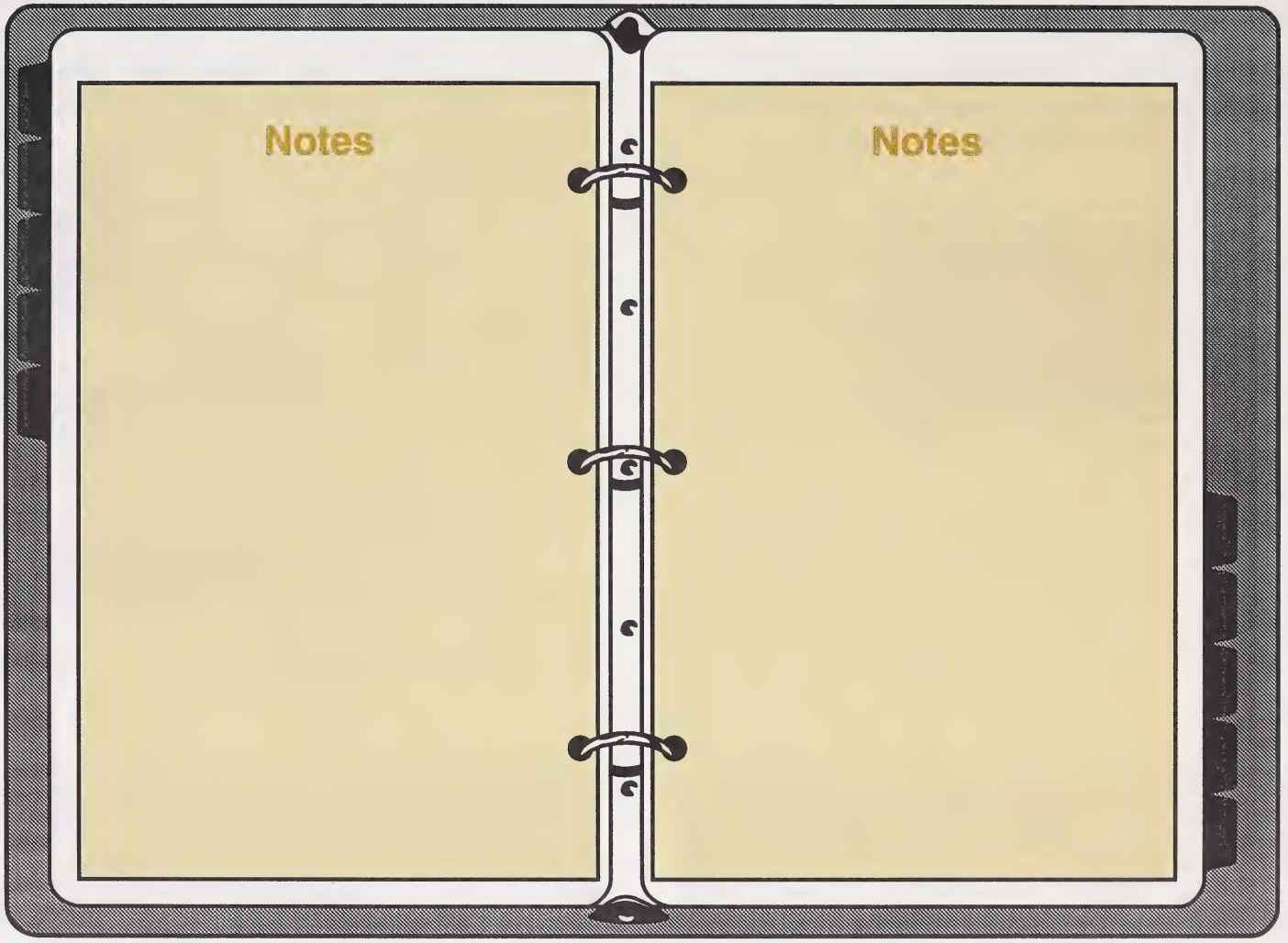
Concluding Activities

Space for Your Work

A pan is made from a rectangular sheet of metal as shown. Squares are cut out of each corner and the sides are folded up. What is the volume of the pan?



See your learning facilitator to check your answers and to receive further instructions.


Notes
Notes

What Lies Ahead

In this section you will review the concepts that you learned in Part 2 of Module 6.

Working Together

At this point it may be a good idea to review the skills that you have learned in Part Two.

Turn to Section 14 and review the Pretest. Then correct any errors you may have made at the time. You may be pleasantly surprised to discover how much you have learned!

What Lies Ahead

The assignment in the Module Conclusion evaluates the achievements of the objectives in this module.

Working Together

Now that you have studied Module 6 and you have done the required practice, you should be ready for the module assignment.

Module Assignment

Turn to the Assignment Booklet and complete the module assignment independently. You may refer to your notes, but do not get help from anyone.

Afterwards, submit the Assignment Booklet for a grade and feedback from your teacher.

APPENDIX

GLOSSARY

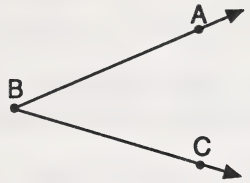
Acute angle: an angle measuring less than 90°

Acute triangle: a triangle with three acute angles

Adjacent sides of a polygon: two sides of a polygon with a common vertex

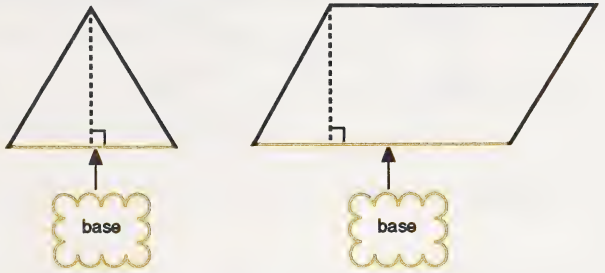
Adjacent vertices: two vertices of a polygon joined by a side

Angle: a set of points consisting of two rays extending from a common endpoint or vertex

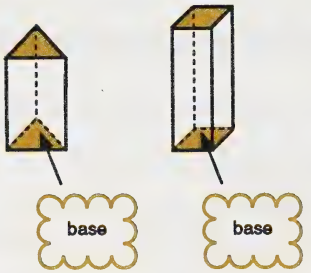


Area: a measurement in square units of the amount of surface a closed curve contains

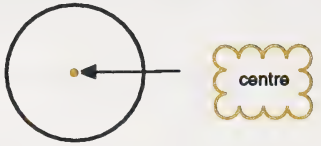
Base of a polygon: the side of the polygon from which the height of the polygon is measured



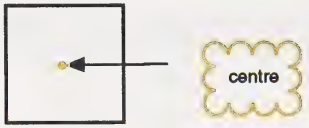
Base of a right prism: the face of the right prism from which the height is measured



Centre of a circle: the point about which the circle is symmetrical



Centre of a polygon: the point about which the polygon is symmetrical



Circumference: the distance around a circle

Closed curve: a connected set of points with no endpoints

Congruent: having the same shape and size

Cube: a right rectangle prism with all six faces congruent

Curve: a connected set of points

Decagon: a polygon with ten sides and angles

Degree of an angle: a unit for measuring angles

Diagonal: a line segment which connects two non-adjacent vertices of a polygon

Diameter: a line segment which joins two points on the circumference of a circle and passes through the centre of the circle

Dodecagon: a polygon with twelve sides and angles

Endpoint: a point at the end of a line segment or ray

Equilateral triangle: a triangle with three congruent sides and angles

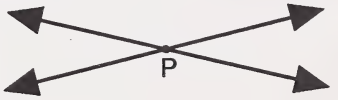
Face: a flat surface on a three-dimensional object

Height of a polygon: the perpendicular distance from the base of the polygon to the opposite vertex

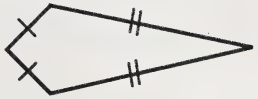
Heptagon: a polygon with seven sides and angles

Hexagon: a polygon with six sides and angles

Intersecting lines: two lines that meet and share a common point



Kite: a quadrilateral with two pairs of congruent adjacent sides

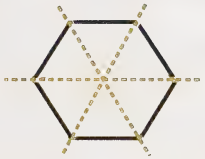


Lateral faces of right prisms: the faces that are not bases of a right prism

Line: a set of points which extends in a straight path infinitely in both directions

Line segment: a part of a line with two endpoints

Line of symmetry: a line that divides a curve into congruent parts that are reflection images of each other



Midpoint: the point which divides a line segment into two congruent parts

Non-adjacent sides: sides of a polygon that do not share a common vertex

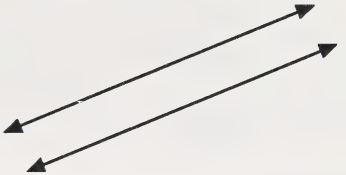
Nonagon: a polygon with nine sides and angles

Non-simple closed curve: a closed curve with crossovers

Obtuse angle: an angle measuring between 90° and 180°

Obtuse triangle: a triangle with one obtuse angle

Parallel lines: two lines in a plane that do not intersect

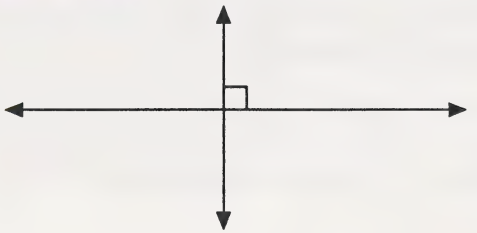


Parallelogram: a quadrilateral with opposite sides parallel

Pentagon: a polygon with five sides and angles

Perimeter: the distance around a closed curve

Perpendicular lines: lines that intersect at right angles



Polygon: a closed curve made of three or more line segments

Octagon: a polygon with eight sides or angles

Quadrilateral: a polygon with four sides and angles

Radius: a line segment that joins the centre of a circle with any point on the circumference

Ray: a set of points that extends in a straight line infinitely in one direction

Rectangle: a parallelogram with 90° angles

Reflex angle: an angle between 180° and 360°

Regular polygon: a polygon with all sides congruent and all angles congruent

Rhombus: a parallelogram with all sides congruent

Right angle: an angle measuring 90°

Right prism: a three-dimensional solid with two congruent bases and rectangular lateral faces

Right rectangular prism: a right prism with rectangular lateral faces

Right triangle: a triangle with a 90° angle

Scalene triangle: a triangle with no congruent sides

Similar: having the same shape but a different size

Simple closed curve: a closed curve with no crossovers

Square: a parallelogram with 90° angles and congruent sides

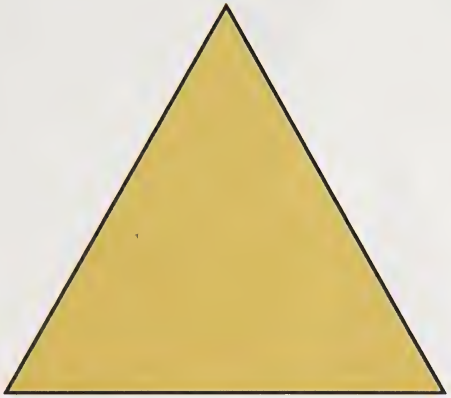
Straight angle: an angle measuring 180°

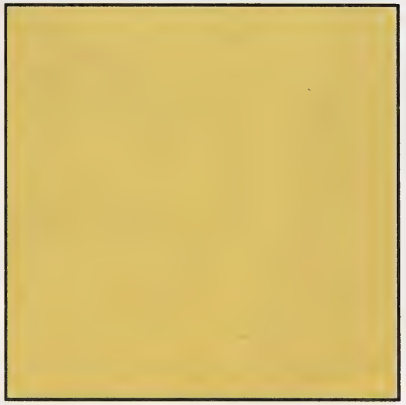
Triangle: a polygon with three sides and angles

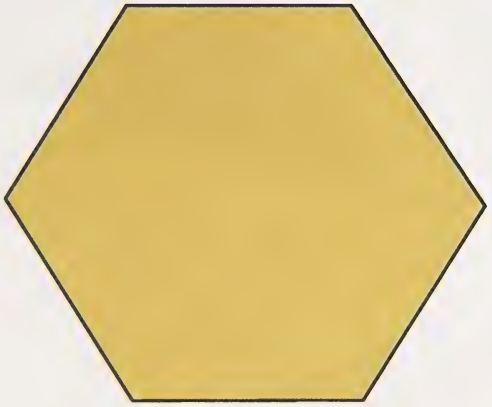
Trapezoid: a quadrilateral with one pair of parallel sides

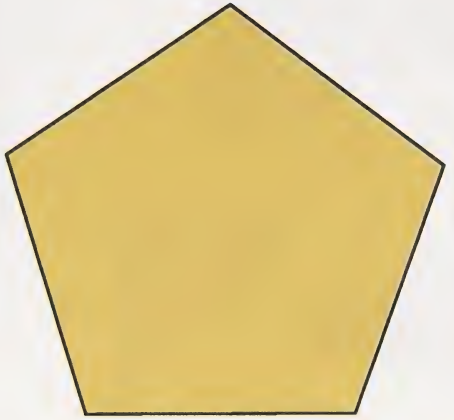
Vertex: the common point of the two rays in an angle, the common point of two sides in a polygon, the common point of two edges in a three-dimensional object

Volume: a measure in cubic units of the amount of space in a three-dimensional object

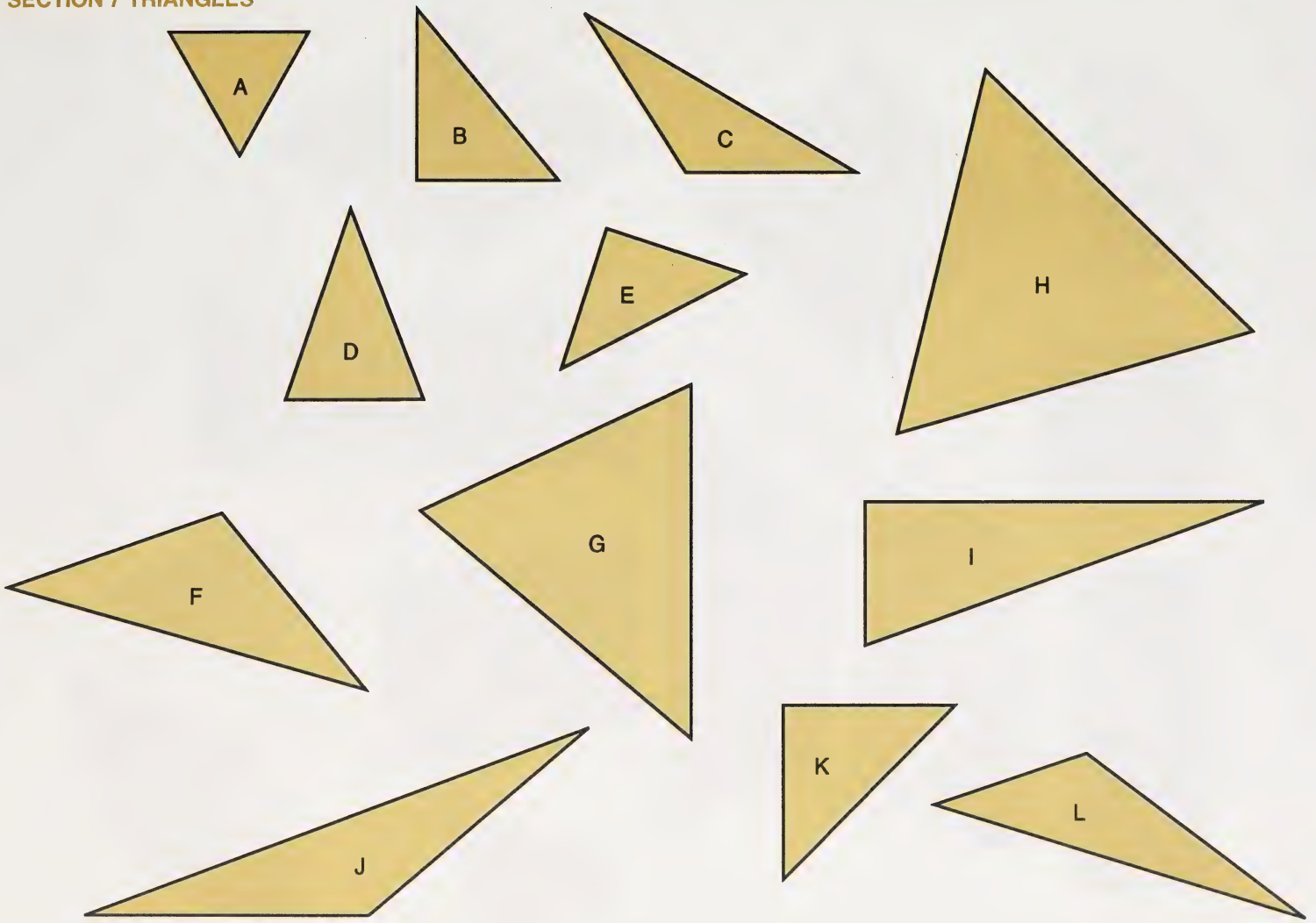




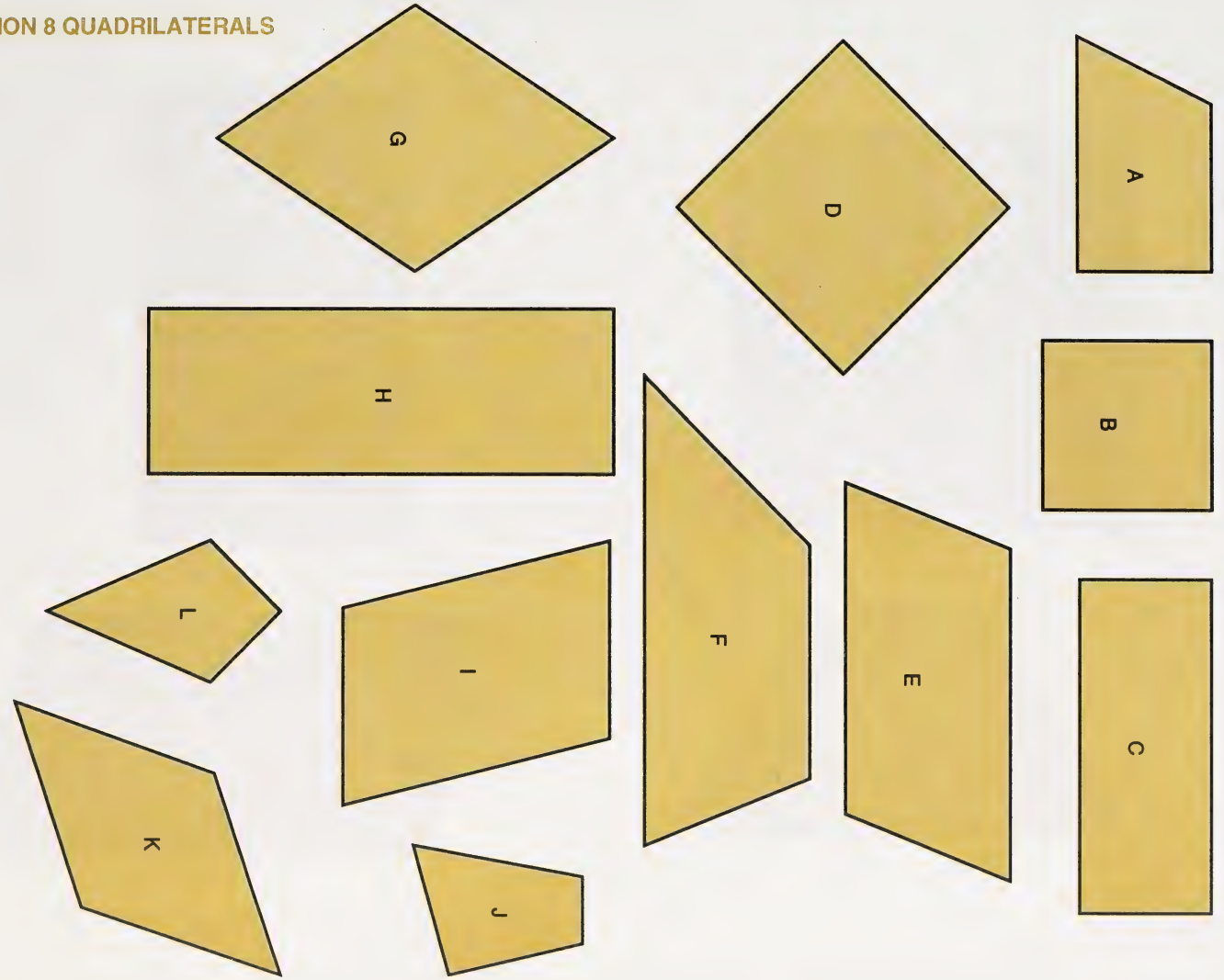




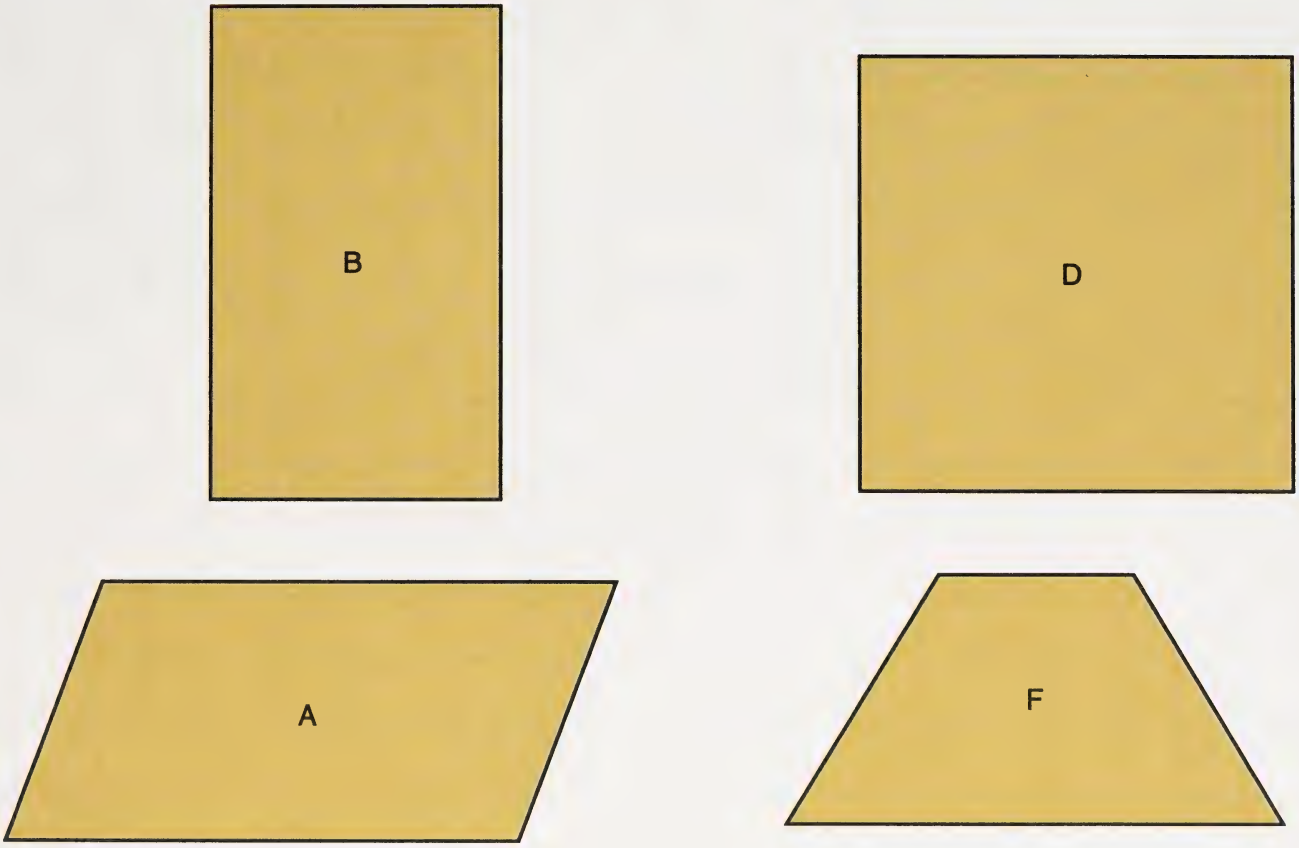
SECTION 7 TRIANGLES



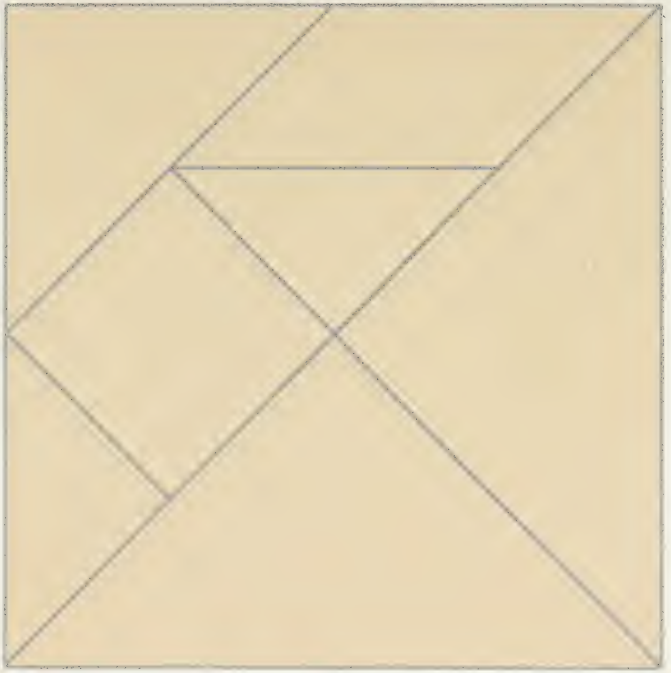
SECTION 8 QUADRILATERALS



SECTION 13 POLYGONS



SECTIONS 6 and 8 TANGRAM



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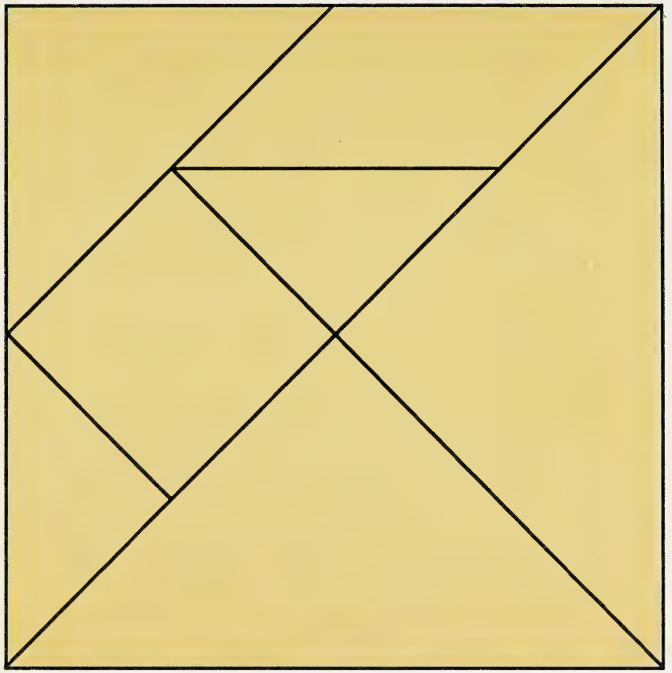

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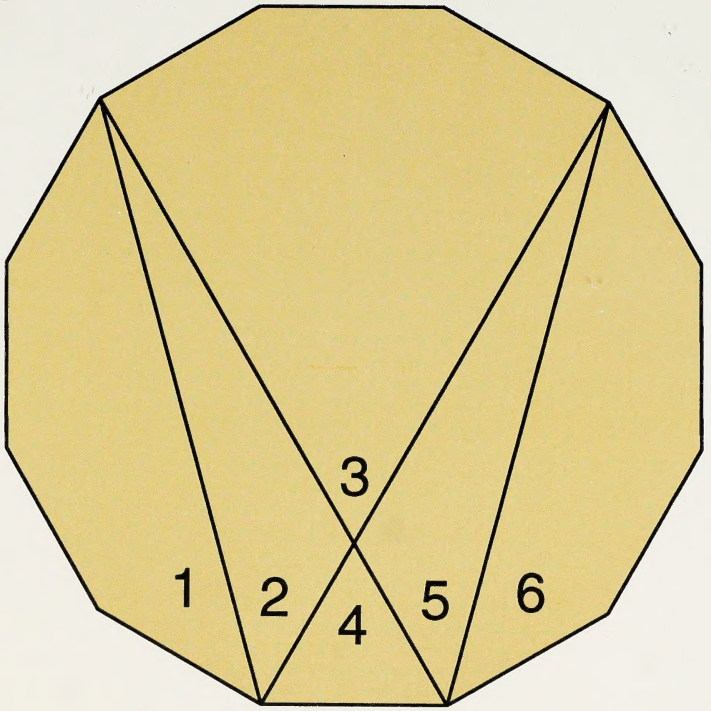

MATH 8

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